Antonine Physics A2

Topic 14B Medical Physics

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This option looks at some of the many ways in which Physics is applied to the medical profession. In the first four tutorials, we look at how we detect sound and vision, which are vital to the way that we interact with the environment. Both of these use important physics concepts. Then we look at how physics is used in imaging techniques that are vitally important to diagnosis of diseases.

Medical Physicists have an important role to play in a modern hospital. Many physics concepts can be applied to the diagnosis and treatment of patients in hospital.

If you are considering a career as a doctor, you will find this option relevant to your studies. Medical Physics stands at the interface of Physics and Biology.

Topic 14 B

Option B Medical Physics

1. Sound and Vision

Tutorial 14B.01 The Eye

AQA Syllabus

Contents

14B.011 Structure of the Eye

The eye is a sophisticated organ. Without it we cannot do many things that we take for granted. Without your eyes you cannot read this text here. We can mimic the action of the eye with a TV camera.

The eye has a structure like this:

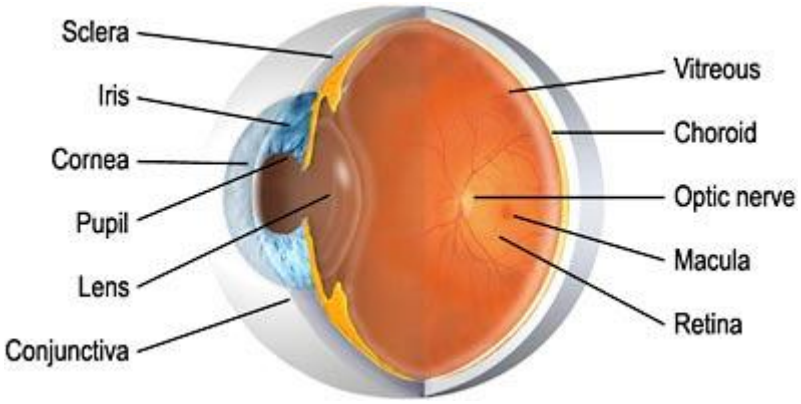

Figure 1 Structure of the eye (Author not known)

In the eye (*Figure 1*):

- The tough outer **sclera** has a transparent region at the front called the **cornea**.
- The muscular **iris** controls the size of the pupil and hence the amount of light reaching the retina.
- The lens is held in position by **suspensory ligaments** and **ciliary muscles**.
- The **retina** contains the **receptor cells** which are sensitive to light.

The eye has a system of muscles called **ciliary muscles** that alter the shape of the lens. If you look at a close-up object the lens fattens, becoming optically stronger. If you look at a distance object the muscles relax and the lens goes thinner (*Figure 2*).

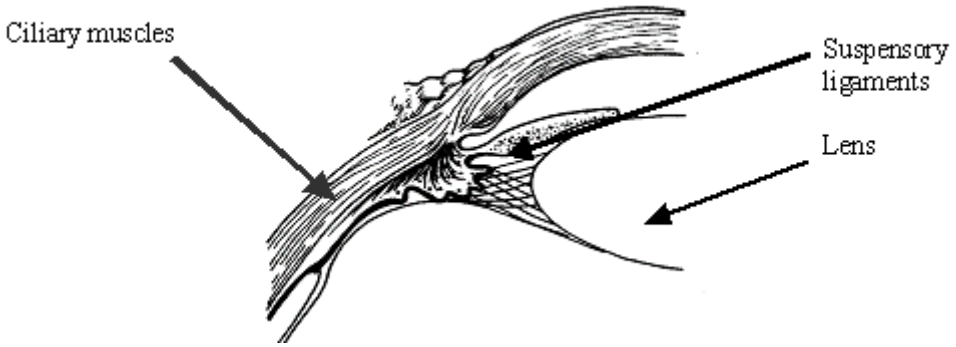

Figure 2 The ciliary muscles

Light from an object enters the eye through the **cornea**. The curved cornea and the lens produce an image on the retina. There are several boundaries at which refraction takes place. However, the **main refraction takes place at the air cornea boundary**, not the lens, which does the fine focusing. The lens does the fine focusing.

Behind the cornea is a watery liquid called the **aqueous humour**. Between the lens and the retina is the **vitreous humour**, a clear jelly like material that helps to keep the eye in shape.

The image is upside down on the retina as shown (*Figure 3*):

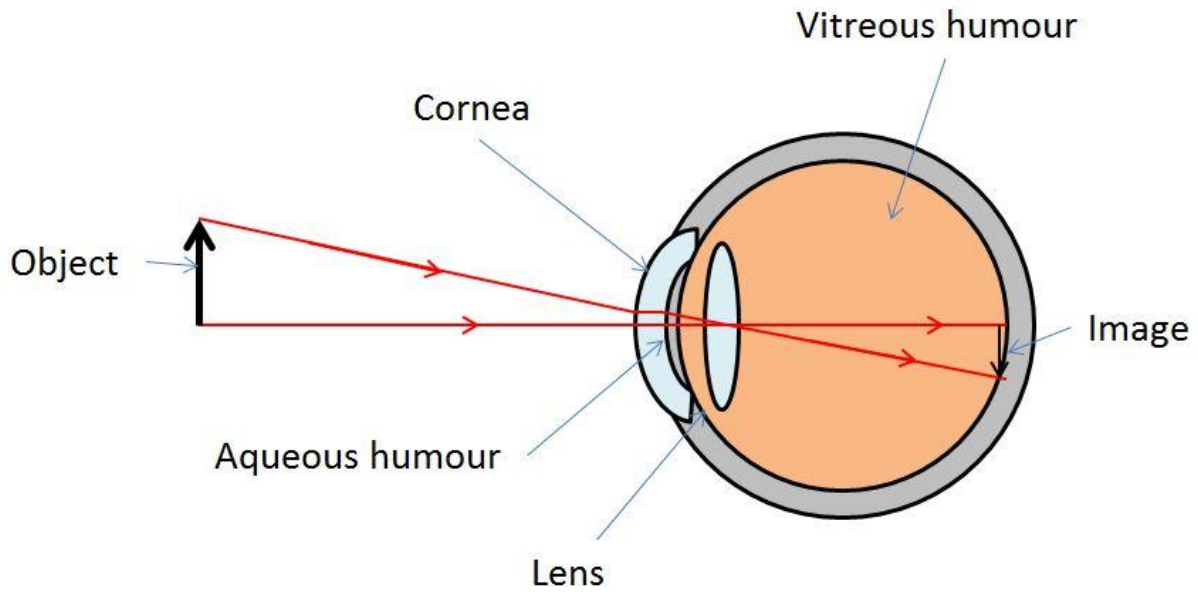

Figure 3 Image on the retina

The image is made on the yellow spot, which is where there is the greatest concentration of cone-shaped light receptor cells (cones).

The retina looks like this (*Figure 4*):

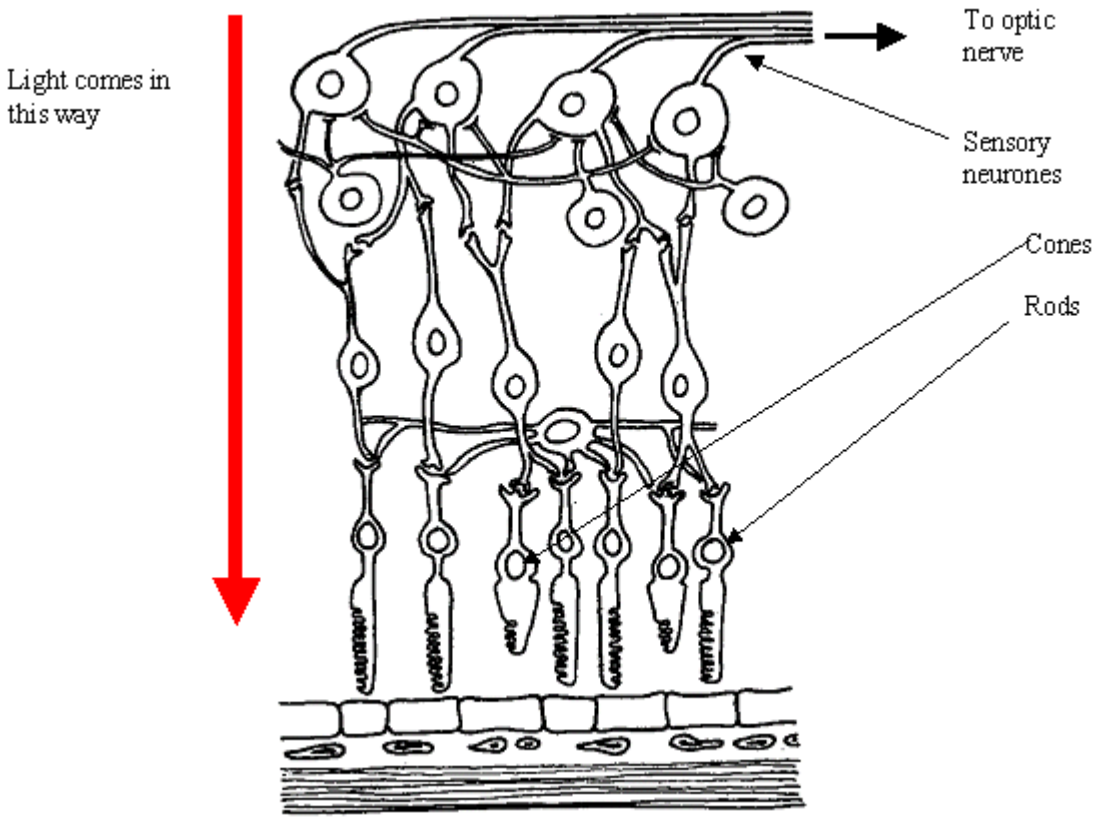

Figure 4 The retina

Notice that the rods and cones are at the **back** of the retina. There are sound biological and evolutionary reasons, but we won't worry about that here.

14B.012 Sensitivity of the Eye

It said that the eye can respond to a single photon. It can respond to a difference in light intensity of 10^9 times. There are two mechanisms:

- the contraction or dilation of the iris
- dark adaptation.

In the dark the iris is dilated. Photosensitive chemicals build up the rod-shaped cells on the retina, a process that can take up to 30 minutes and is called **adaptation**. Rods also share nerve cells, so smaller intensities cause a signal. However, the visual acuity (how accurately you can see) is reduced. Also, the rods are spread either side of the yellow spot. Therefore, the image is not well focused.

The rods are most sensitive to light of wavelength about 510 nm (green light), while the sensitivity of cones as a whole is about 560 nm (yellow light). This can be shown in the graph below (*Figure 5*).

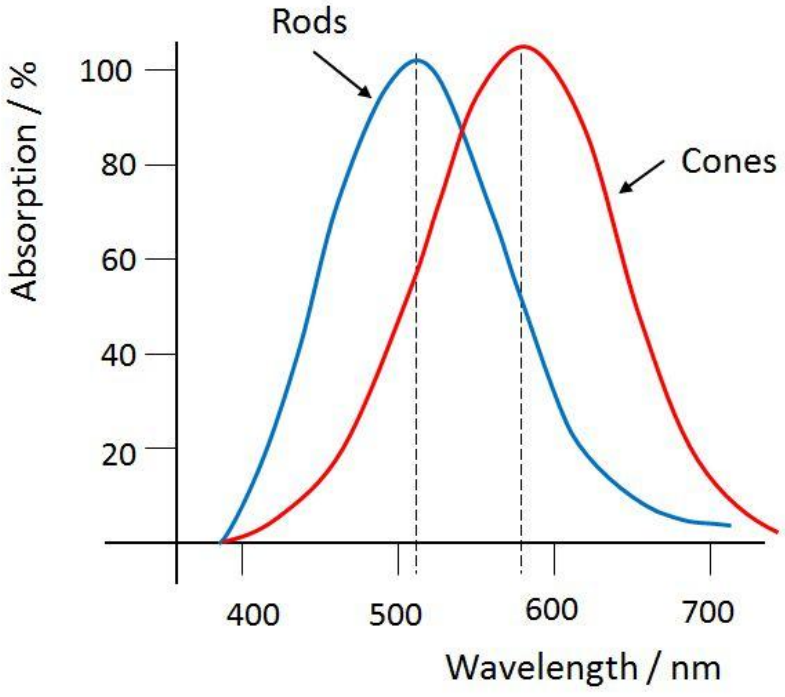

Figure 5 Sensitivity of rods and cones to wavelength of light

In low light levels, when only the rods are operative, we see in monochrome (black and white). A certain level of light is needed before colour vision is possible.

In low levels of light, the iris dilates. In bright light, the iris constricts. It is like adjusting the aperture (the f-stop) of a camera. The image is focused onto the yellow spot where there are large numbers of cones. There are three kinds of cones:

- Cones sensitive to red light.
- Cones sensitive to green light.
- Cones sensitive to blue light.

The colour that is seen depends on the proportion in which each type of cone is stimulated. The most sensitive cones are the green ones, while blue cones are fairly insensitive. The graph (*Figure 6*) shows the way in the cones respond to different wavelengths.

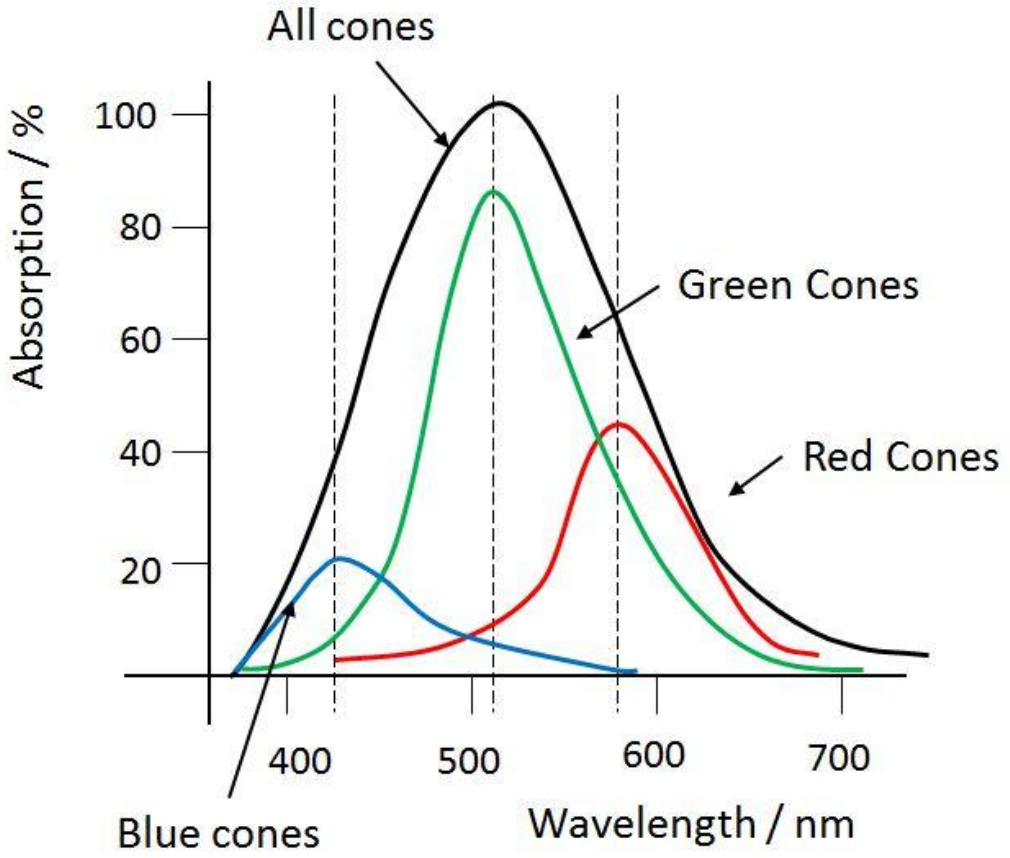

Figure 6 Wavelength sensitivities of different kinds of cones

On the retina the spacing of cones is about 3 μm apart, which gives an **angle of resolution** (which represents the **visual acuity**) of about 0.008° which is quite small. In

the rod rich area of the retina the acuity is about 1/20th of the acuity of the yellow spot, about 0.16°.

Nerve ends respond to **changes in light level**. The eye is never still; it is constantly **scanning** so that new nerves are stimulated. However, the nerve endings take a definite time to respond. Flickering lights fuse into a continuous light. This is why you don't see the flickering of a TV set. The delay period is called **persistence of vision**. Movies are filmed at 24 frames a second; the eye can fill in the gaps between the action of each individual photograph.

A frog's eyes are fixed. It cannot see the insect in front of it until it moves. Then it can aim with good accuracy.

A flash light for a camera lasts for less than 1 ms. We see it for longer due to the persistence of vision.

14B.013 Depth of Field

If you take a picture with a camera, a large aperture gives a reduced **depth of field**. So, if you focus on a near object, far objects are out of focus. If you reduce the aperture, you can bring the far objects into sharper focus without changing the position of the lens. The downside is that detail can be lost.

It's the same with the eye. If you are focused on a near object in bright light, you will be able to see far objects reasonably clearly. If you are focused on the same object at the same distance in dim light, far objects are blurred. The depth of field is reduced. See *Figure 7*.

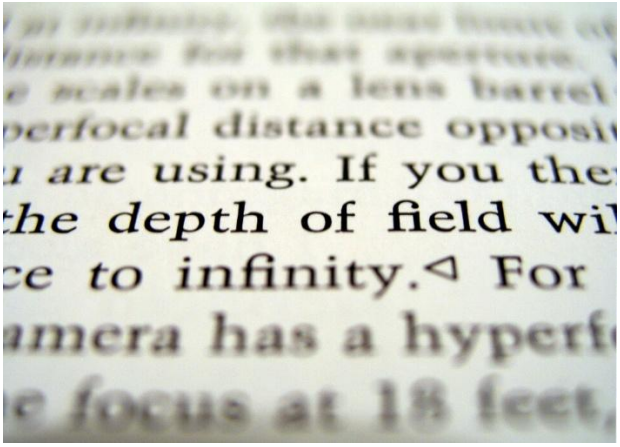

Figure 7 Depth of Field (PiccoloNamek, Wikimedia Commons)

Questions

Tutorial 14B.01

14B.01.1

List three different ways in which the eye is like a TV camera. How is the eye superior to a TV camera?

14B.01.2

"Bad light stopped play". Cricket games in the UK are often abandoned if the weather gets gloomy. Can you explain why?

14B.01.3

Explain why yellow is a good colour for a high-visibility jacket.

14B.01.4

Explain how the eye changes the depth of field.

Tutorial 14B.02 Defects of Vision and their Correction

AQA Syllabus

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14B.021 Convex Lens

Lenses work by **refracting** light at a glass-air boundary. Although refraction occurs at the boundary, we will treat all lenses as bending the rays at the **lens axis**. You may want to go back to **Topic 7** to revise refraction.

The lens in the eye is a **convex** or **converging lens**. This means that the lens makes rays of light come together or converge (*Figure 8*).

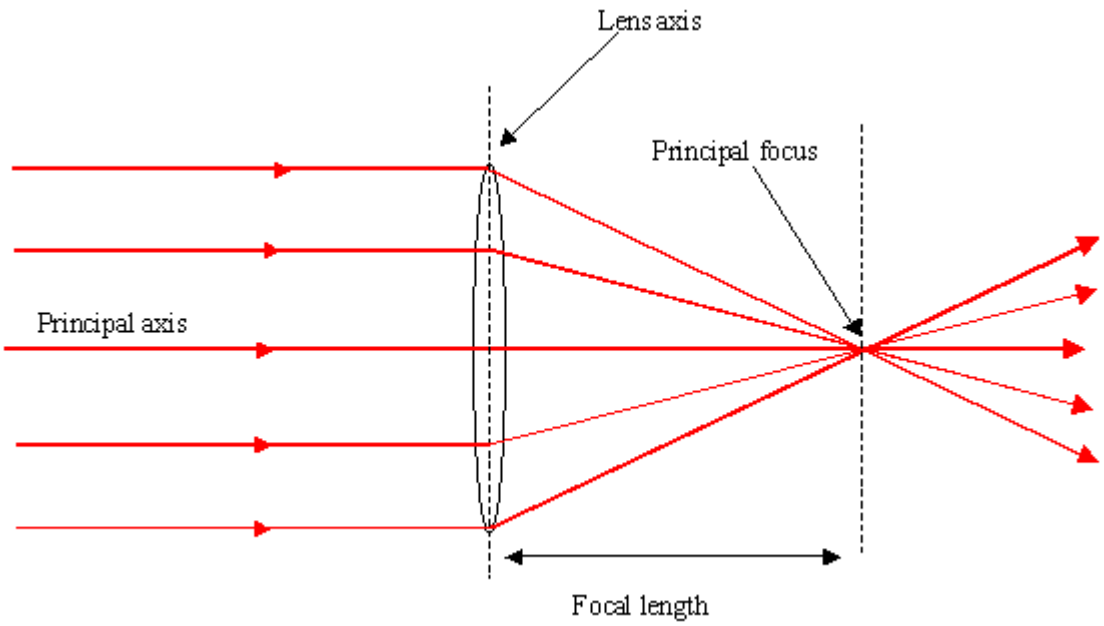


Figure 8 Ray diagram for a converging lens

The rays parallel to the **principal axis** are converged onto the **principal focus**. The **focal length** is the distance between the **lens axis** and the **principal focus** (strictly speaking, the focal plane).

14B.022 Power of a Lens

Thicker lenses bend light more and are therefore described as more powerful. Powerful lenses have short focal lengths. The **power** of a lens is measured in **dioptres** (D) and is given by the formula:

$$\text{Power} = \frac{1}{\text{focal length (m)}}$$

In code:

$$P = \frac{1}{f} \quad \text{..... Equation 1}$$

If two lenses in optical contact, their powers add up:

$$P = P_1 + P_2 \quad \text{..... Equation 2}$$

The principal focus of a convex lens is called **real**. The images made by convex lenses are in most cases real. This means that the image can be projected onto a screen. We will see later how images are made with **ray diagrams**.

Principal, NOT principle.

Principal means “chief”, or “main”.

Principle means a rule.

14B.023 Concave Lens

The concave lens splits rays parallel to the principal axis, which is why it is called a **diverging lens** (*Figure 9*).

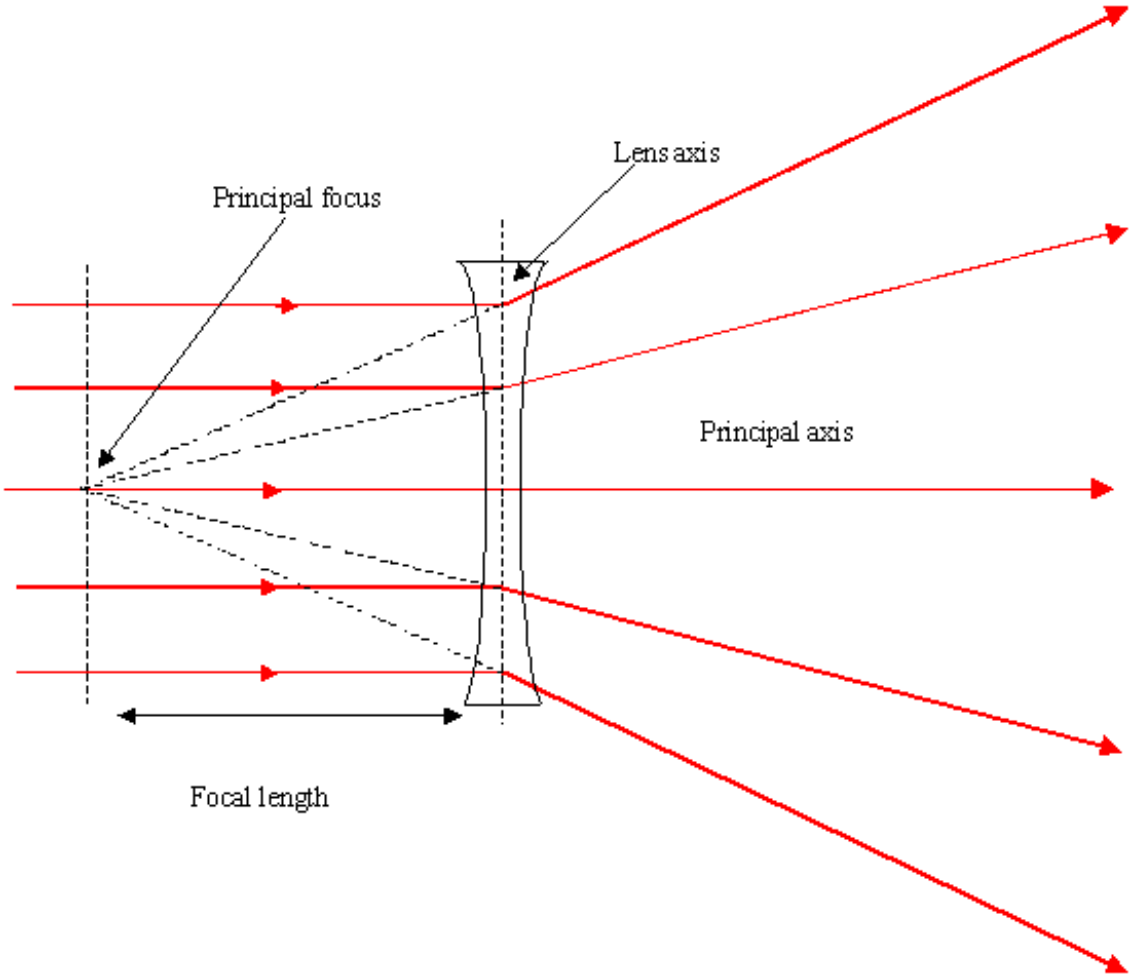


Figure 9 Ray diagram of a diverging lens

The **principal focus** is virtual because the rays do not pass through it but diverge as if they had come from it. Images in concave lenses are always **virtual** because they cannot be projected onto a screen.

The power of a concave lens is always **negative**, for example -1.5 D.

14B.024 Ray Diagrams

We can determine where an image lies in relation to the objects by using a **ray diagram**. We can do this by using two simple rules:

- Draw a ray from the top of the image parallel to the principal axis. This ray bends at the lens axis and goes through the principal focus.
- Draw a ray from the top of the lens through the centre of the lens.

Where the two rays meet, that is where the image is found. The diagrams shows how we do a ray diagram step-by-step:

Step 1 Draw the ray parallel to the principal axis (*Figure 10*).

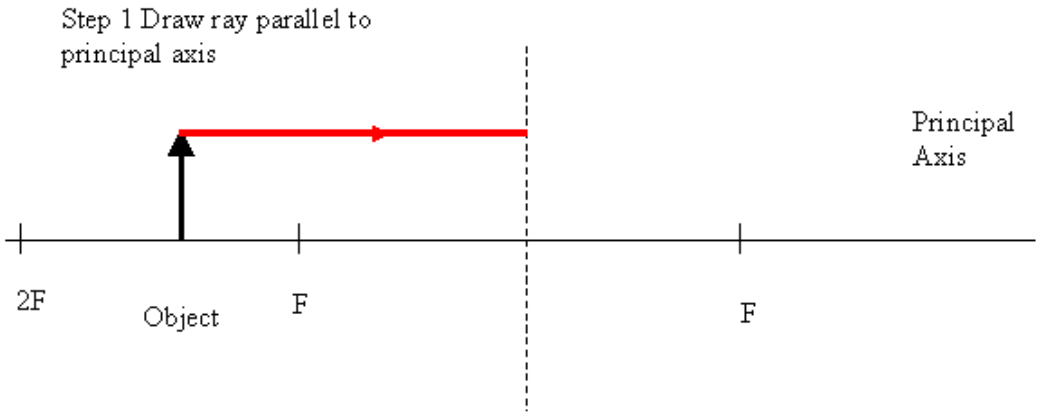


Figure 10 Step 1, draw the parallel ray

Step 2 Draw the refracted ray so that it passes through the principal focus (*Figure 11*).

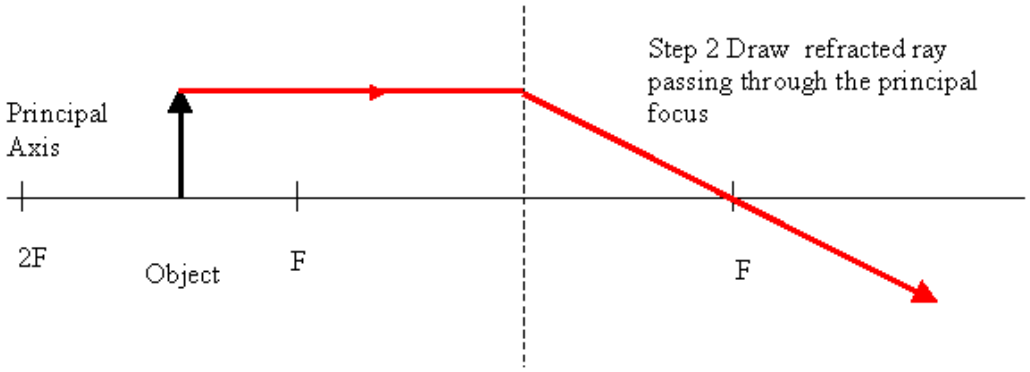


Figure 11 Step 2, draw the ray through the principal focus

Step 3 Draw a ray from the top of the object through the middle of the lens. This ray is **undeviated** (*Figure 12*).

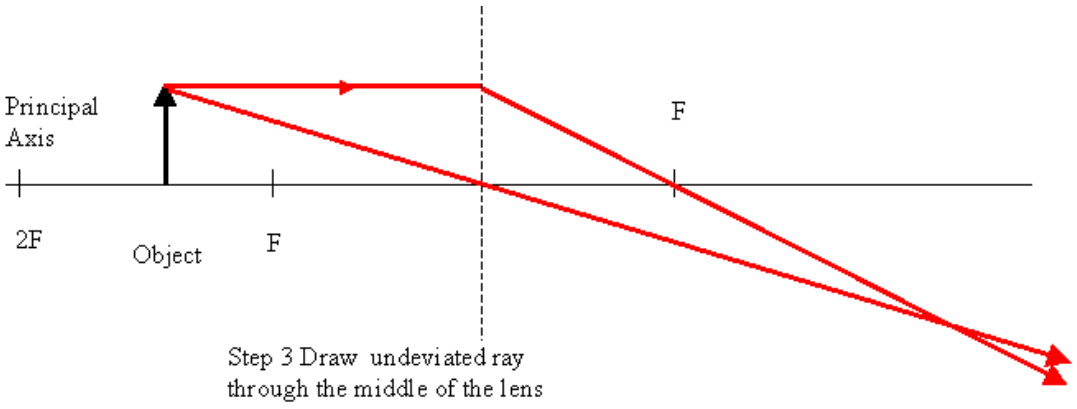


Figure 12 Step 3, drawing a ray from the top of the object through the middle of the lens

Step 4 Where the rays meet, that is where the image is (*Figure 13*).

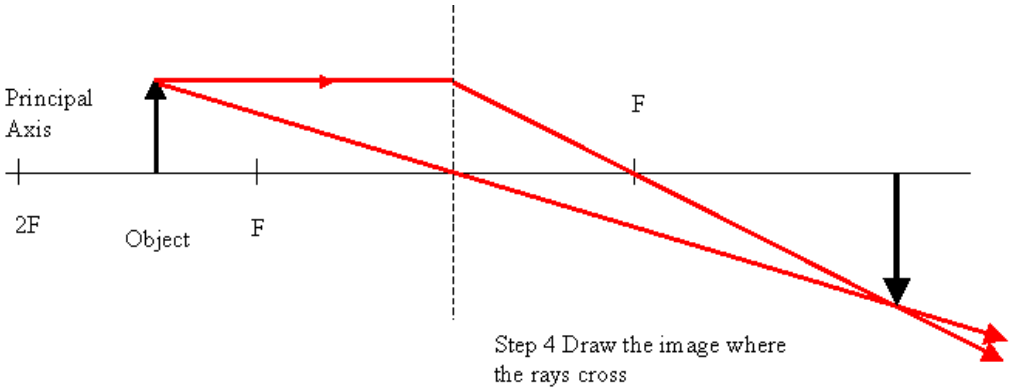


Figure 13 Step 4, completing the ray diagram

It is a good idea to draw your ray diagrams on graph paper as the following ray diagrams are. Be careful with your drawing; a small change in the angle of the undeviated ray can lead to quite a big change in the final position of the image. And PLEASE… Be a good chap and use a sharp pencil. *Figure 14* shows a ray diagram done on graph paper.

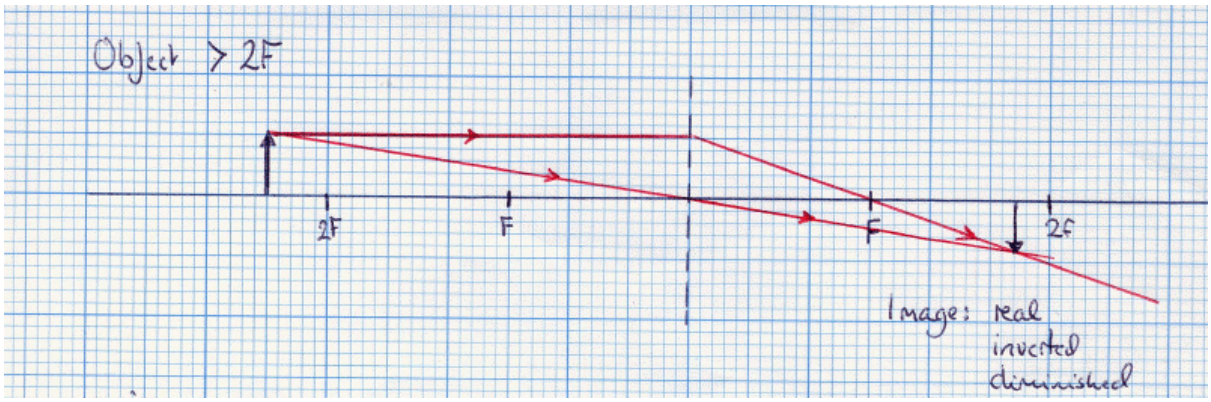


Figure 14 Ray diagram drawn on graph paper

This diagram shows where an object is at a distance of greater than twice the focal length. The image is **inverted** (upside down), **real**, and **diminished** (smaller).

For a **concave** lens, the process is similar, except that we extend the refracted parallel ray back to the virtual principal focus (*Figure 15*).

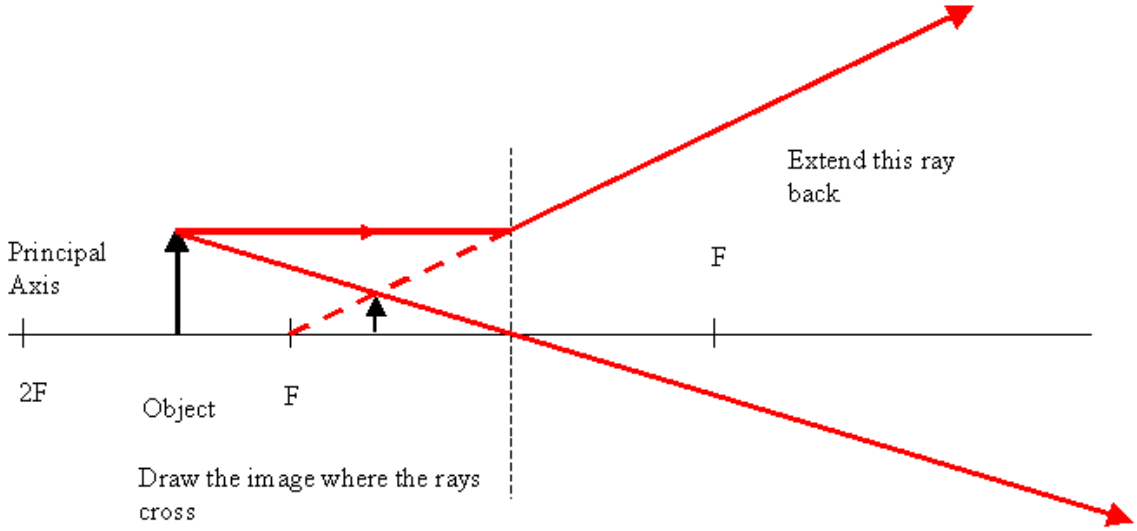


Figure 15 Ray diagram for a diverging (concave) lens

14B.025 The Lens Formula

Lens diagrams have the main disadvantage that there is uncertainty in precisely where the image is. Therefore, the use of the **lens formula** is better. The lens formula is:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

..... *Equation 3*

[f - focal length (m); u - object distance (m); v - image distance (m)]

Worked Example

An object of height 1.6 cm is placed 50 cm from a converging (convex) lens of focal length 10 cm. What is the position of the image?

Answer

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Substitute:

$$\frac{1}{10\ \text{cm}} = \frac{1}{50\ \text{cm}} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{10\ \text{cm}} - \frac{1}{50\ \text{cm}} = 0.10\ \text{cm}^{-1} - 0.02\ \text{cm}^{-1} = 0.08\ \text{cm}^{-1}$$

v = 1 ÷ 0.08 cm^{-1} = **12.5 cm**

It does not matter if you work in cm, as long as you are consistent. However, if you are going to use **dioptres** you must work in **metres**.

The **magnification** is worked out using this simple formula:

$$M = \frac{v}{u}$$

..... *Equation 4*

Since v is in metres, and u is in metres, M has no units.

Worked Example

What is the magnification in the example above? What is the size of the image?

Answer

M = 12.5 cm ÷ 50 cm = 0.25

Image height = 1.6 cm × 0.25 = 0.40 cm = 4.0 mm

The convention for the equation is that **real is positive**. For a concave lens, the focal length is negative, because the principal focus is virtual. If the image position gives a negative value, then the image is **virtual**.

14B.026 Defects of Vision

The optician uses lenses to correct defects of vision. There are two defects that we will think about here:

- **Myopia** (short sight), where the cornea is curved too much, or the lens is too powerful. The power of the eye is too great, and the image of far objects is formed in front of the retina (*Figure 16*). Far objects appear blurred.

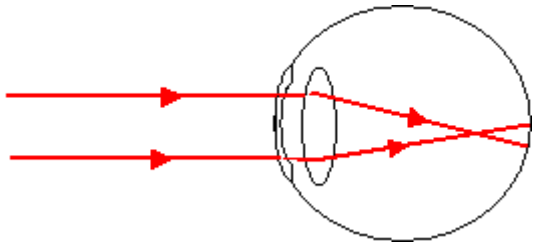


Figure 16 Myopia (short sight)

- **Hypermetropia** (long sight) is where the eye is not powerful enough, leading to the image of near objects is behind the retina (*Figure 17*). Near objects appear blurred.

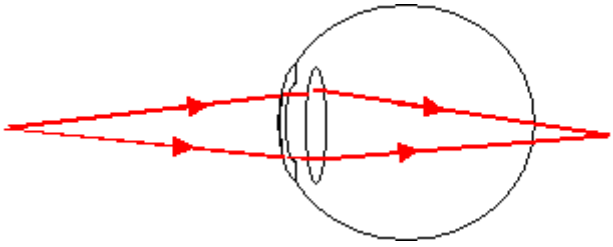


Figure 17 Hypermetropia (long sight)

Short sight is corrected by using a concave (diverging lens) to make the parallel rays move apart. The optician will find the **far point**, which is the furthest distance that the unaided eye can focus a clear image. To correct short sight, the far point needs to be at **infinity**. The power of the lens can be worked out:

Power of the corrective lens = Power of the corrected eye - power of the uncorrected eye

$$P_{\text{lens}} = P_{\text{corrected}} - P_{\text{uncorrected}} \quad \text{..... Equation 5}$$

Worked Example

The far point of a patient is 60 cm. If the distance from the lens to the retina is 2.0 cm, what is the power of the eye? What is the power of the eye with a corrective lens? What is the power of the corrective lens? What kind of lens is it and what is its focal length?

Answer

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{0.60\ \text{m}} + \frac{1}{0.020\text{m}}$$

The power of the eye at the far point is:

$P = 1/u$ = 1/0.60 m = 1.67 D

Power of the lens focusing light onto the retina is:

$P = 1/v$= 1/0.020 m = 50 D

Power of the short-sighted eye is:

$P = 1/f$ = 1.67 m^{-1} + 50 m^{-1} = 51.67 D

For the corrected eye:

$$P = \frac{1}{\infty} + \frac{1}{0.020\ \text{m}} = 50\ \text{D}$$

Power of the corrective lens = Power of the corrected eye - power of the uncorrected eye

Power = 50 - 51.67 = **-1.67 D**

Since the power is negative, the lens must be **concave** (diverging).

Its focal length is **0.60 m**

The diagram (*Figure 18*) shows how short sight can be corrected using a diverging lens.

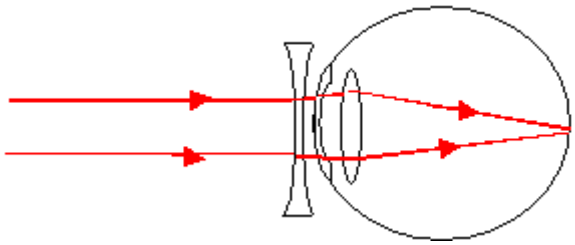


Figure 18 Correcting short sight

Spectacle lenses are not biconcave as shown, as they look odd. Instead, the front is slightly convex, and the back is much more concave. It's a matter of taste, you know...

The **near point** is the point that the eye can focus onto a close up object. The eye is made more powerful by the lens being made fatter. Generally, the eye can change power by about 5 dioptres.

Worked Example What would the near point be of the patient in the previous worked example on page 19, assuming the eye has a power change of 5 D?
Answer At its most powerful, the eye would have a power of 56.67 D Power = 57.67 D = $1/u$ + 1/0.020 m = $1/u$ + 50 D $1/u$ = 7.67 D u = **0.13 m**

With long sight, a converging lens is used to make the eye more powerful. The corrective lens is shown in the diagram below (*Figure 19*).

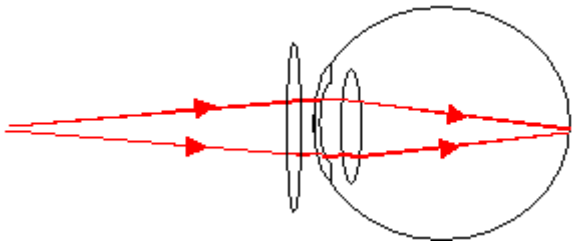


Figure 19 Correcting long sight

Astigmatism is a condition where the cornea is not spherically curved. It has different curves in different directions, so that images are sharper in one direction than another. The result of this is vision that is blurred or distorted in any direction. It can lead to eyestrain, headaches, and being unable to drive at night. Its causes are not clear, but it is thought to have a genetic element.

Lenses are made to compensate for the differences. Astigmatism can also be combined with short or long sight. The idea is shown below (*Figure 20*).

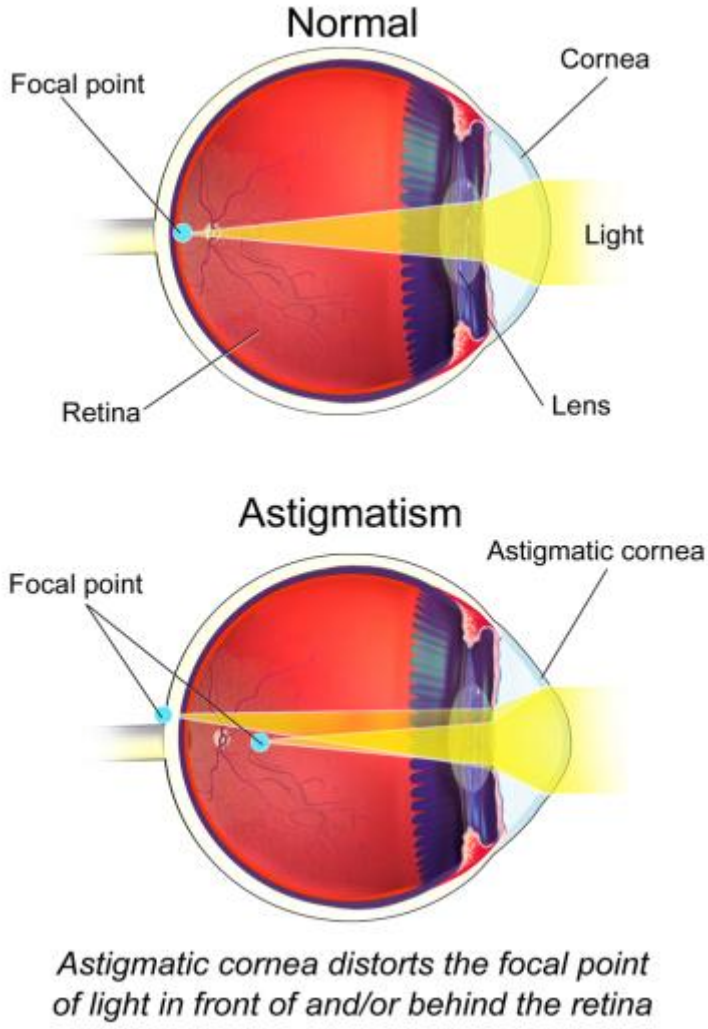


Figure 20 Astigmatism (Image by BruceBlaus - Wikimedia Commons)

Treatment for astigmatism can be:

- Glasses.
- Contact lenses.
- Surgery that permanently changes the shape of the retina.

Contact lenses can be placed directly on the eye. They correct in exactly the same way as the lenses for spectacles. **Monocles** are single lenses to correct the defect in one eye only. They are associated with the stereotype of an upper-class twit and are nowadays a prop for period drama.

Questions

Tutorial 14B.02

14B.02.1

The optician's prescription for a lens is +0.2 D. What is the focal length in metres?

14B.02.2

What is the image like if the object is at 2F?

14B.02.3

What is the image like if the object is between 2F and F?

14B.02.4

What is the image like if the object is at F?

14B.02.5

What is the image like if the object is less than F?

14B.02.6

What is the image like for a concave lens?

14B.02.7

Find the position and size of a pound coin, 2.2 cm in diameter placed 20 cm from a converging lens of focal length 40 cm.

14B.02.8

The same coin is now placed 20 cm in front of a diverging lens of focal length 40 cm. What is the position and the size of the image now?

14B.02.9

What is the change in focal length represented by 5 dioptres?

14B.02.10

Look at the worked examples on Pages 19 and 20.

What is the near point of the patient's eye in the examples above with corrective lenses on?

14B.02.11

A patient has a near point of 3.0 metres. Assuming that the distance from the front of the eye to the retina is 2 cm,

(a) what is the power of her eye unaided?

(b) What power of eye will she need to read a book at 25 cm?

(c) What power should the lens be and what kind of lens is it?

(d) What is the focal length?

Tutorial 14B.03 The Physics of Hearing	
AQA Syllabus	
Contents	
14B.031 Acoustic Quantities	14B.032 The Ear
14B.033 Anatomy of the Ear	14B.034 Frequency Response of the Ear
14B.035 Intensity Response	14B.036 The Decibel Scale
14B.037 Hearing Loss	

Sound is a **longitudinal wave**. It travels as a series of compressions and rarefactions. We know that in air the speed = 336 m s^{-1}. If you are not sure about the nature of longitudinal waves, revise waves in Waves Tutorial 2. Sound also travels in other materials. In steel the speed is 6000 m s^{-1}. In water sound travels at 1500 m s^{-1}. There is formula that can be used to relate the speed of sound to the density and the bulk modulus of a solid. You are not expected to know it for the exam.

14B.031 Acoustic Quantities

Some materials conduct sound better than others. We can use an analogy to electrical resistance. We use the term **acoustic impedance**, which is the product of the density and the speed of sound in the material. A material with low acoustic impedance conducts sound well. One with a high acoustic impedance conducts sound badly. The equation is given below:

$$Z = \rho v$$

..... *Equation 6*

Z - acoustic impedance (kg m^{-2} s^{-1}); ρ - density (kg m^{-3}); v - speed (m s^{-1}).

At the boundary of two materials with widely differing acoustic impedances, most of the sound is reflected and little is transmitted. This has important implications in ultrasound scanning.

The **intensity** of a sound is the **power per unit area**. The further you are away from any source, the lower the intensity. Like all progressive waves the intensity decreases with an inverse square law relationship. Double the distance and the intensity drops to a quarter.

Intensity is given the Physics code *I* and is measured in watts per square metre (W m^{-2}).

Materials with a high acoustic impedance attenuate sound waves, which means the intensity is reduced.

14B.032 The Ear

The **ear** is an amazing piece of kit. About the size of a matchbox, it contains a high fidelity sound detection system that can pick up a huge range in intensity. It can be tuned to be sensitive to very quiet sound or adapt to exposure of very loud sounds. It can discriminate particular sounds above a whole cacophony of other noise. No electronic instrument has the capabilities of the ear. Added to that, it is also important for maintaining balance.

We have two ears (really?) for a purpose. Not only do two ears allow us to hear the sounds but also give important spatial information about the direction of the sounds and the layout of the sounds. This had important evolutionary consequences. We could tell which direction a predator was coming from. If we listen to a well set-up stereo system, we can tell the positions of all the instruments in the orchestra (*Figure 21*).

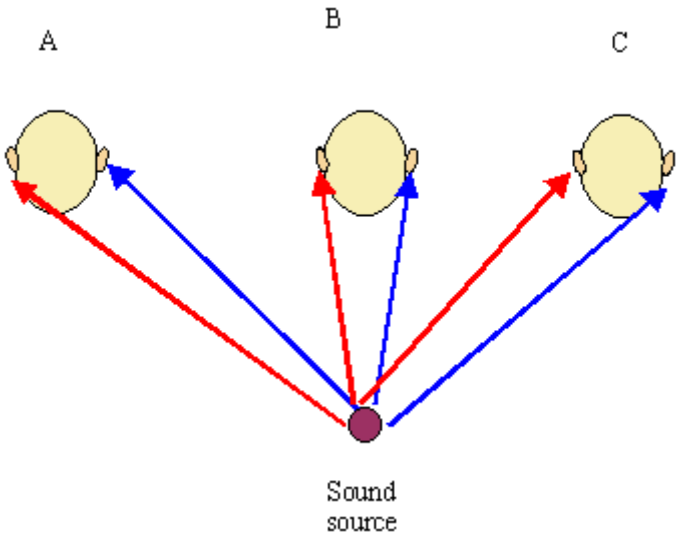


Figure 21 Listening position when listening to a stereo system

The ear can discriminate a time difference of about 10^{-5} s.

In many animals the **pinna** of the ear (the flapper) can be moved about to funnel the sound into the ear to get a more accurate location. In the human, the muscles are too weak to move the ear, but electrical activity can be detected if a sound is made. This is a test that doctors use on babies to test their hearing.

14B.033 Anatomy of the Ear

The structure of the ear is shown below (*Figure 22*).

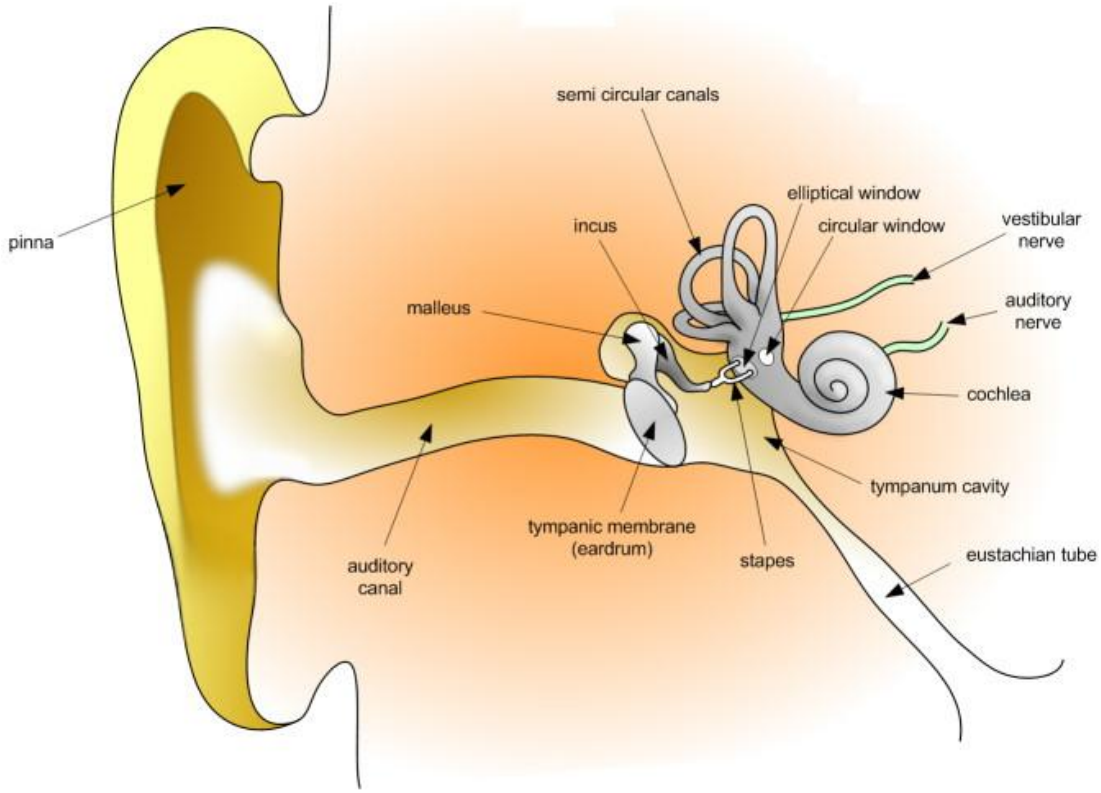


Figure 22 Anatomy of the ear (Graphic by Dan Pickard, Wikimedia Commons)

- The **pinna** acts as collecting device that funnels the sound waves into the ear canal.
- The **ear canal** increases the intensity of the sound by reducing the area.
- The sound causes the **tympanic membrane** to vibrate.
- This vibrates the **ossicles** (the three small bones);
- Which in turn strike the **oval window** of the **cochlea**.
- The **cochlea** has nerve cells that detect the sound to convert it into electrical impulses for processing by the brain.

The ossicles are three small bones that amplify the sound by increasing the movement through a system of levers (*Figure 23*). Their names are:

- **Malleus** (hammer)
- **Incus** (anvil)
- **Stapes** (stirrup)

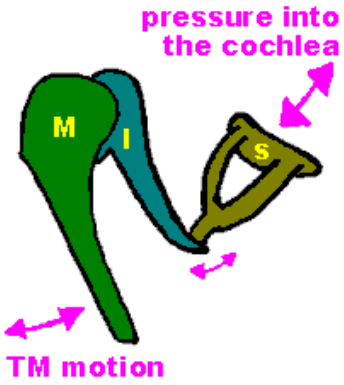


Figure 23 The ossicles

However, in loud environments there are muscles that act to restrict the movement of the ossicles and reduce the amplification effect. This helps to avoid damage. The muscles take time to react, and very sudden intense sounds can do major damage to the ear.

The cochlea is a fluid filled canal which is lined with **hair cells**. The sound impulses are sent through the fluid to be vibrate the hair cells. The picture below (*Figure 24*) shows the hair cells.

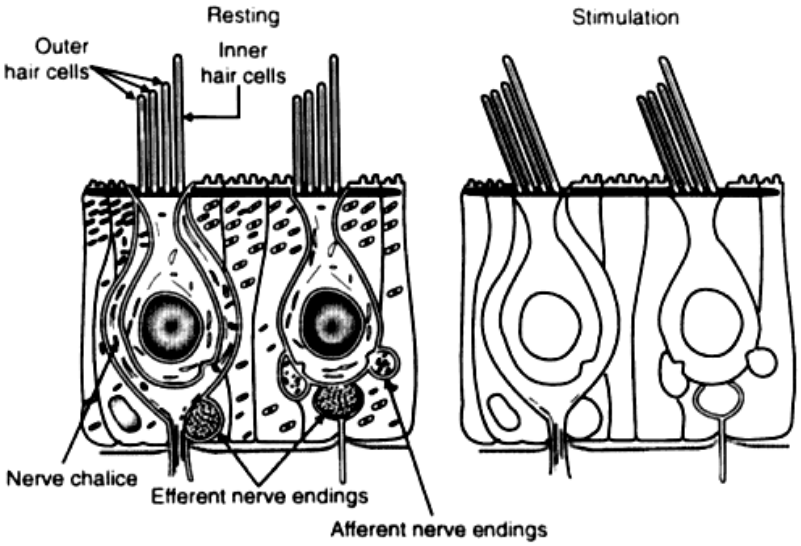


Figure 24 Hair cells in the lining of the cochlea

There are different theories as to how the sounds of different frequencies are picked up in the ear. These are summed up in the picture below (*Figure 25*):

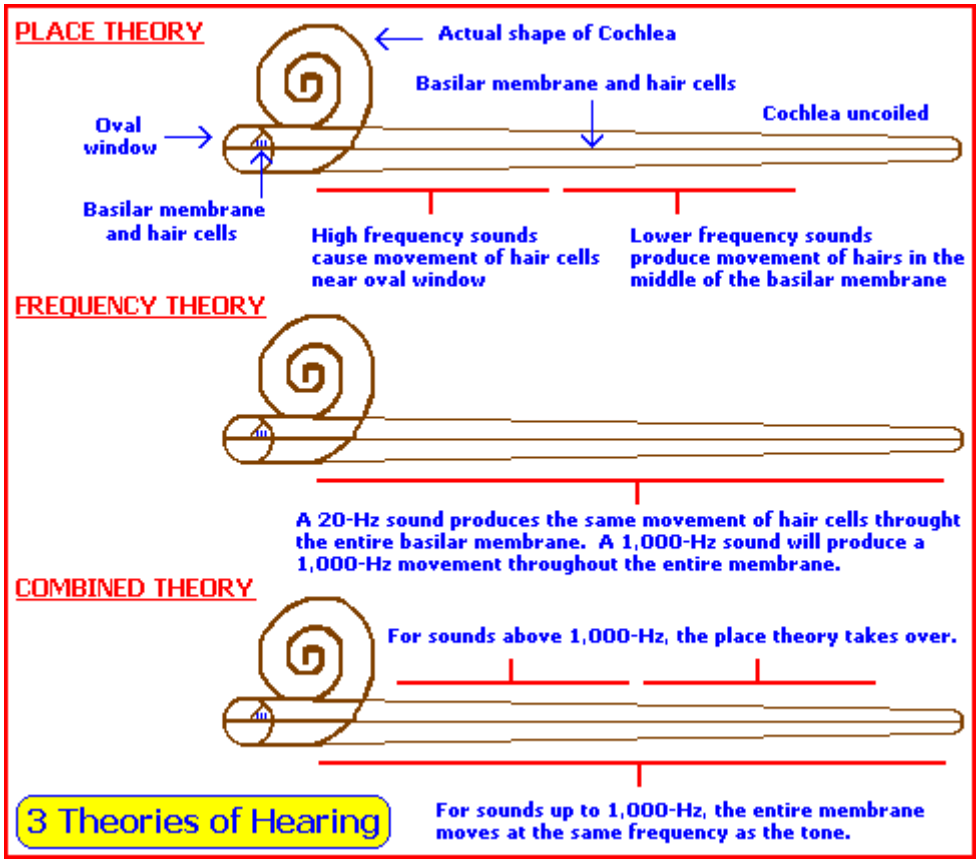


Figure 25 Theories on how different frequencies are detected by the cochlea

14B.034 Frequency Response of the Ear

At the lowest, the human ear can detect sounds of frequency about 20 Hz. Sounds below this frequency are called **infrasound** and are felt rather than heard. In a young person, the upper limit is about 20000 Hz, although the upper limit comes down with age. A middle aged person will hear frequencies up to about 15000 Hz, where an elderly person has an upper limit of about 10000 Hz.

The human ear has a peak sensitivity of 3000 Hz, which causes a sense of unease. The human scream is at this frequency, and alarms are designed to sound at 3000 Hz. The reason for this is that the cochlea has a tube length of about 2.5 cm, closed at one end and open at the other.

The frequency response of the ear is shown below (*Figure 26*):

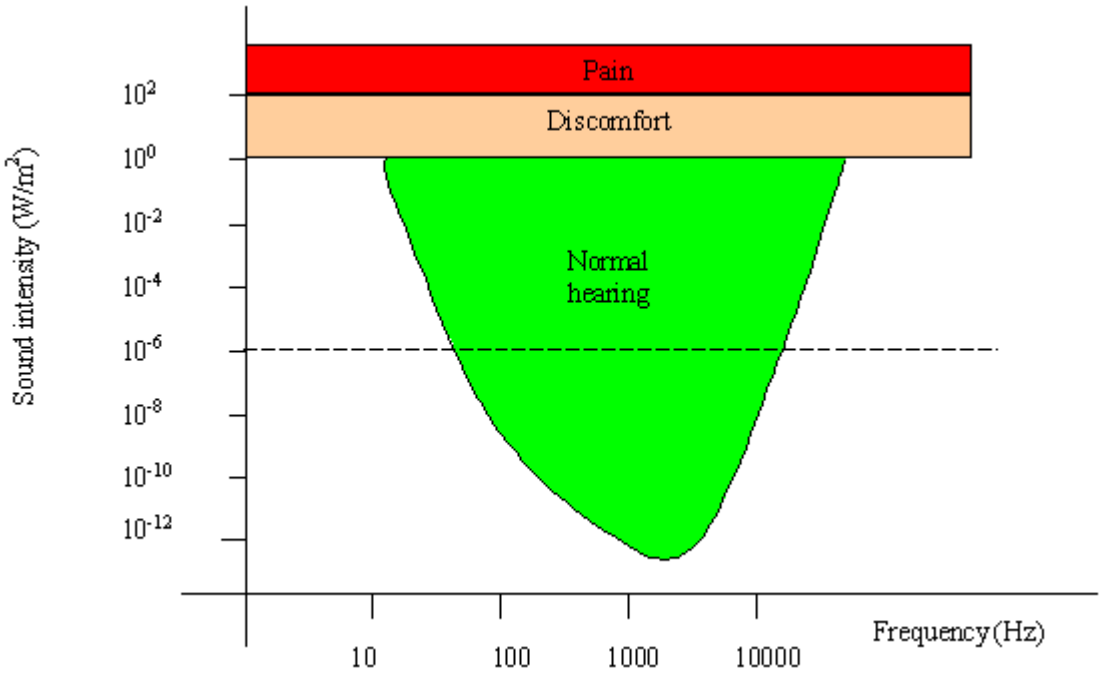


Figure 26 Frequency response of the human ear.

Notice that:

- At low frequencies the ear is very insensitive. The intensity of sound needed at 20 Hz is about 1 watt per square metre (Very loud).
- As the frequency goes up, the **threshold of hearing** gets rapidly less. At 100 Hz the intensity needed to hear a sound is 10^{-10} W m^{-2}.
- The ear has a very low threshold of hearing for 3000 Hz. A sound of this frequency is very penetrating.
- The graph itself has **logarithmic scales**. A logarithm is a number expressed as a power of 10. For example, 100 is 10^2 and 200 is $10^{2.3010}$. It is a useful way of compressing long graphical axes.

The graph here (*Figure 27*) shows the sensitivity of the ear to a fixed reference level:

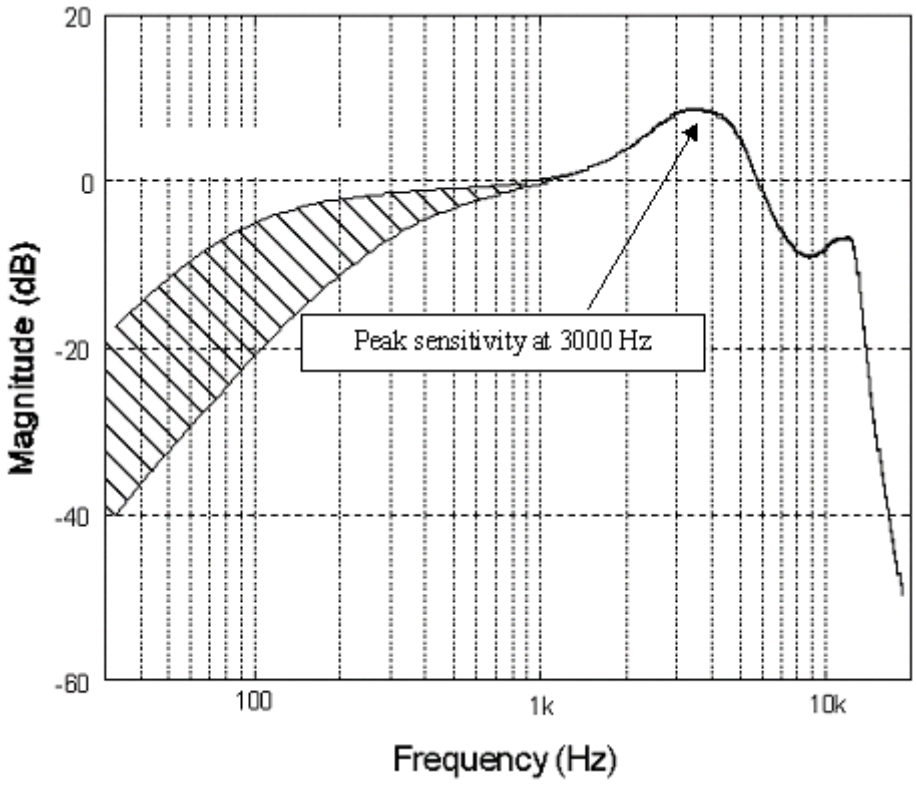


Figure 27 Sensitivity of the ear at a fixed sound level

The ear can discriminate the difference between frequencies. Above 10 000 Hz this ability is poor. In the range 60 - 1000 Hz, a change of 3 Hz can be detected. Certain ratios of frequency are particularly pleasing to the ear, and these form the basis of music.

14B.035 Intensity Response (Loudness)

The loudness of a sound depends not just on the intensity, but also of the energy transfer characteristics of the ear. Loudness is measured in units called **phon**. The graph below (*Figure 28*) shows how the ear responds to sounds of equal loudness, but of different frequencies.

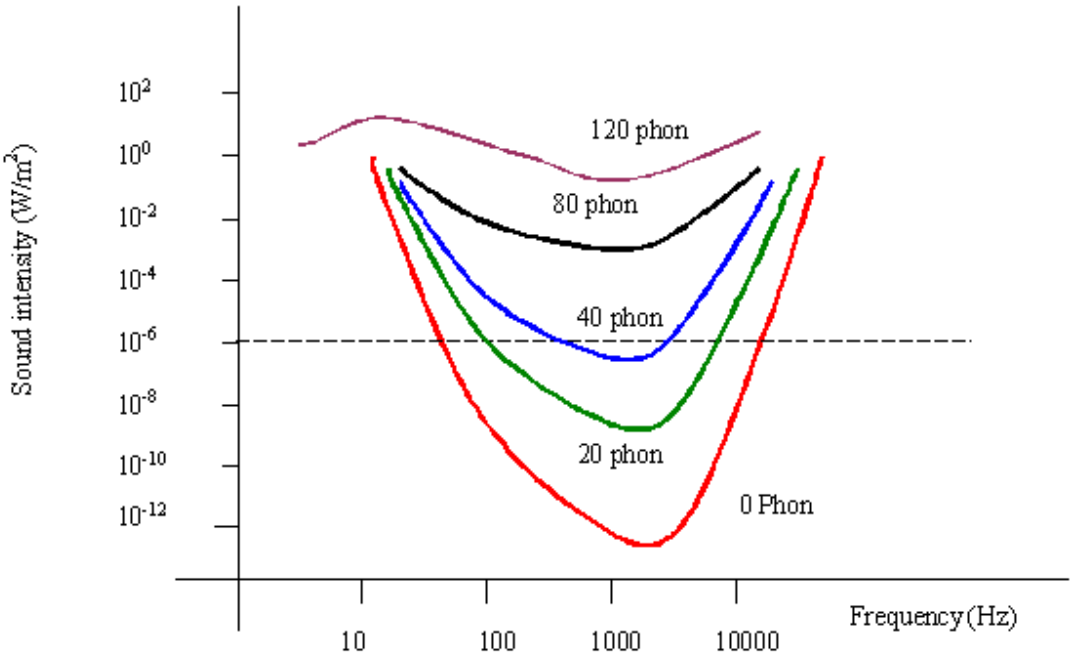


Figure 28 Frequency response of the ear at different sound levels

The key point is that the **perception of changes in loudness does not correspond directly to the change in intensity**. There is a strong frequency element in this. Small changes in intensity can be detected at 3000 Hz, but not at 100 Hz.

Measuring the frequency response to the ear is **very subjective**; you cannot dismantle somebody's head and connect the ear to a CRO.

14B.036 The Decibel Scale

The range of intensities we have seen is huge and we use logarithmic scales to compress the graph into something more manageable. In the decibel scale we use the minimum threshold of hearing, 1×10^{-12} W m^{-2} as a reference point, and we give the reference point the physics code I_0. We can write an expression for the change in intensity ΔI.

$$\Delta I = \log_{10} \frac{I}{I_0}$$

..... *Equation 7*

1 Bel is the intensity change from 10^{-12} to 10^{-10} W m^{-2}. The Bel (B) is rather a big unit, and we use the **decibel** (dB) instead.

$$\Delta I = 10 \log_{10} \frac{I}{I_0}$$

..... *Equation 8*

Worked Example

The maximum limit that is acceptable in a noisy environment is 10^{-3} W m^{-2}. What is this in decibels?

Answer

Use

$$\Delta I = 10 \log_{10} \frac{I}{I_0}$$

$\Delta I = 10 \times \log (10^{-3}\ \text{W m}^{-1} / 10^{-12}\ \text{W m}^{-2}) = 10 \times \log (10^{9})$

$\Delta I =$ **90 dB**

A **doubling** of intensity gives a **3 dB** increase. The ear can just about detect this.

The **dBA scale** is used to take into account the frequency dependence. Remember that the ear is most sensitive at frequencies between 100 to 10 000 Hz. The table shows the levels of certain noises:

Level (dBA)	*Noise*	*Effect*
0	Threshold of hearing	
20	Blood pulsing	
30	Ticking watch	
40	Quiet conversation	
50	Quiet street	
70	Hoover in a room	
90	Road drill at 7 m	Prolonged exposure can lead to hearing damage
100	Noisy factory	
120	Loud discothèque	Threshold of discomfort
140	Aircraft at 25 m	Threshold of pain
160	Rifle close to ear	Ear drum ruptured

14B.037 Hearing Loss

Loss of hearing can be a result of:

- mechanical damage due to a blow on the head.
- disease.
- exposure to excessive noise.

Disease can stop the ossicles from moving. This can be corrected by surgery. Alternatively, a hearing aid can be used that transmits the vibrations through the bones of the skull. Loss of ear sensitivity can be compensated for by a hearing aid. If disease destroys the **nerve fibres** to the cochlea, then nothing can be done. The cochlea is surrounded by bone and very difficult to get to.

Tinnitus is the term given for a continuous ringing or hissing in the ear. This can be a temporary effect after a loud rock concert or can be a long term and distressing condition where the patient cannot sleep.

Serious hearing damage can result from prolonged exposure to loud noises. **Noise** is not easy to define, as one person's pleasant sounds may be a nightmare to others. Some noises can be ignored at first but if repeated often enough can lead to stress. A man in Middlesbrough was murdered a few years ago by a neighbour infuriated by his DIY activities.

The Health and Safety at Work Act now makes it mandatory that ear defenders are used by employees using noisy machinery. However, that can be undone if the employee wears a personal stereo on the way to work that can be heard by the whole train carriage. Excessive noise can break off the hears of the hair cells. The graph (*Figure 29*) shows the hearing of a normal 40 year old and one that has been exposed to a loud environment.

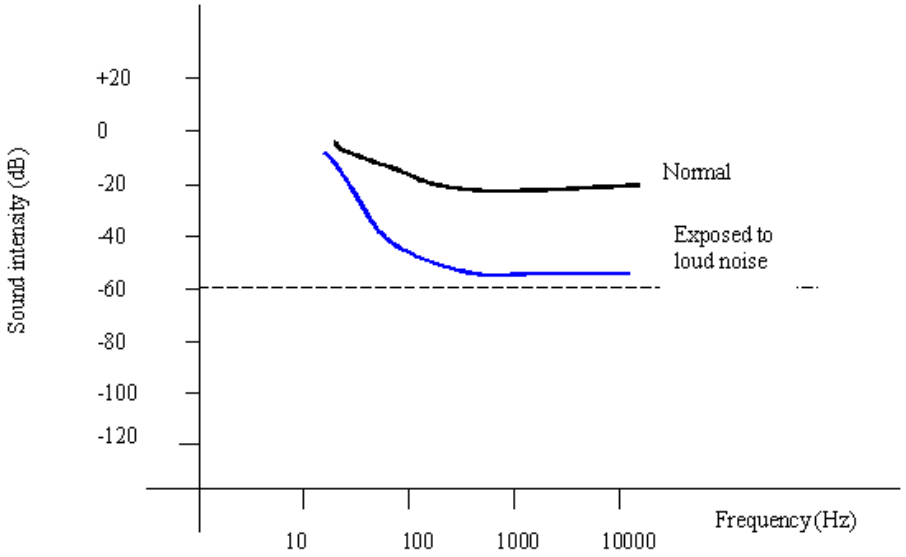


Figure 29 Hearing loss in a 40-year-old exposed to loud noise

Aging also leads to hearing loss. The graph (*Figure 30*) here shows the hearing loss of a 65 year old compared to a 40 year old. Neither has been exposed to prolonged periods of excessive noise.

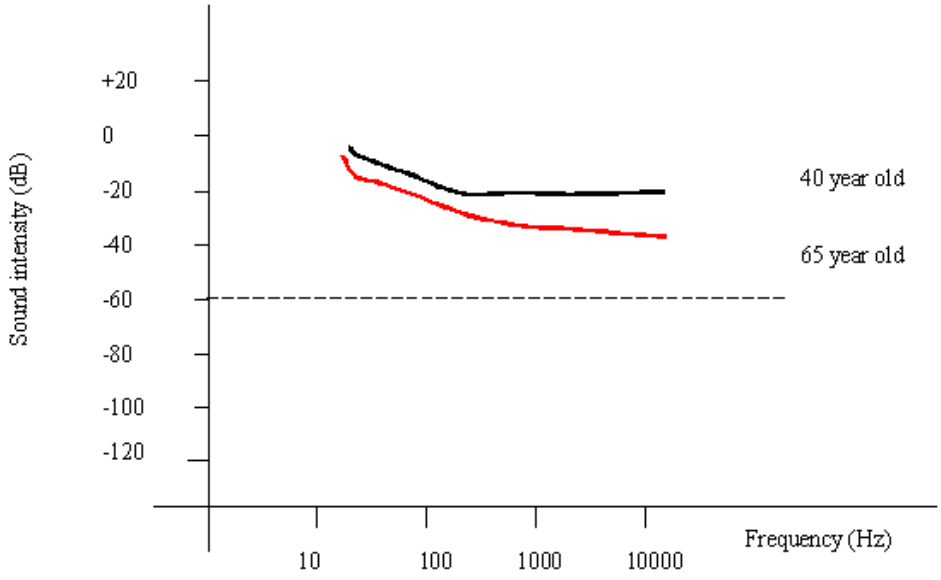


Figure 30 Hearing loss of a 65-year-old compared with a 40-year old (neither exposed to excessive noise)

If the 65 year old had been exposed to **excessive noise** the response line would be much lower. Notice that the higher frequencies are much more affected.

Questions

Tutorial 14B.03

14B.03.1

Look at *Figure 21* on page 26.

Explain how each of the listeners A, B, C hear the sound from the sound source the way they do.

14B.03.2

What path difference does a time difference of 1.0×10^{-5} s represent if the speed of sound in air is 340 m s^{-1}?

14B.03.3

Look at page 29.

Show that the standing wave formed in such a system has a wavelength of about 10 cm.

14B.03.4

See *Figure 26*.

What frequency does the ear resonate at?

14B.03.5

Refer to *Figure 27*. What is:

(a) the intensity of the minimum threshold of hearing and what range does this occur over?

(b) the intensity of the discomfort and pain. Are they frequency dependent?

(c) the audible frequency range at an intensity of 10^{-6} W m^{-2}?

14B.03.6

The sound intensity levels in a school classroom vary from 30 dB to 80 dB. What is the change in sound intensity as a ratio?

14B.03.7

You are walking in the countryside when you are suddenly "buzzed" by a jet aeroplane flying at 25 metres. What is the change in intensity in decibels? Assume that the sound level in the countryside is 30 dB.

14B.03.8

Sketch a graph showing the hearing response of a 40 year old office worker compared to a 65 year old who has worked with noisy machinery all his life.

Tutorial 14B.04 Nerve Impulses and the Heart	
AQA Syllabus	
Contents	
14B.041 Electrical Signals in the Body	14B.042 Nerve Cells
14B.043 The Heart	14B.044 The Electrocardiogram

14B.041 Electrical Signals in the Body

Electrical signals are carried about the body by nerve cells. All cells have a membrane potential because on the inside of the membrane there are more potassium ions, and on the outside, there are more sodium ions. The potential is maintained by a biological mechanism called the **sodium potassium pump** (*Figure 31*).

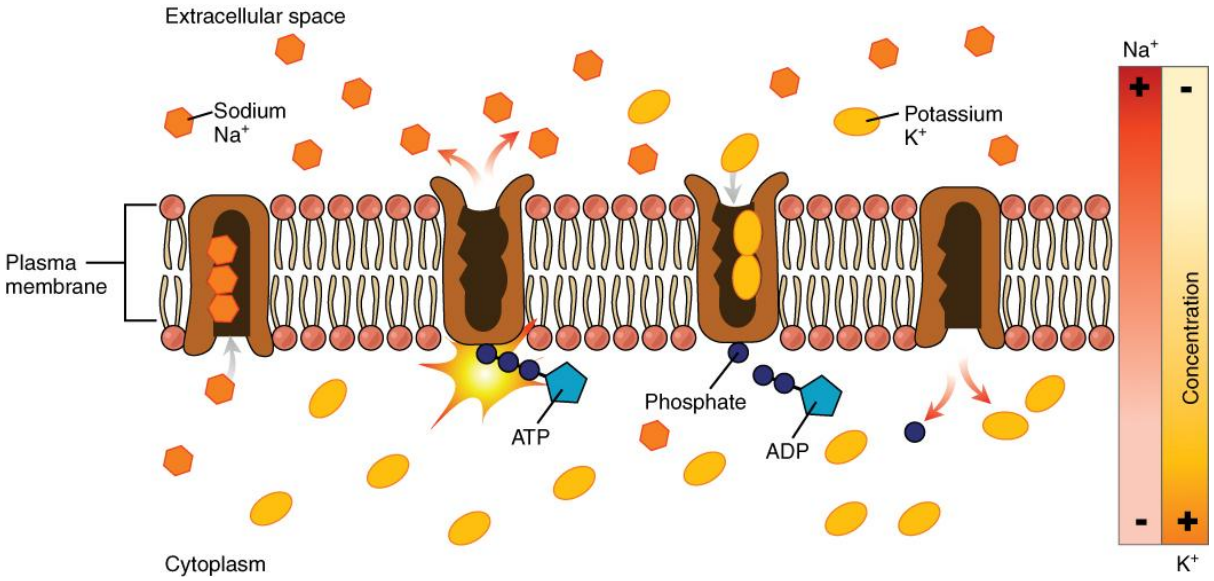


Figure 31 Sodium potassium cell membrane pump (Image from Wikimedia Commons. https://cnx.org/contents/FPtK1zmh@8.25:fEI3C8Ot@10/Preface)

The membrane potential difference is about 70 mV, with the outside being regarded as being at 0 and the inside being at -70 mV. The negative ions are carried on large organic ions that cannot cross the cell membrane. When a cell membrane on a nerve cell is stimulated,

- it suddenly becomes permeable to sodium ions which diffuse through, attracted by the negative charge.
- The potential rises initially to 0 millivolts (depolarisation)
- and then to +30 mV (reverse polarisation).

- Then the membrane becomes impermeable to sodium ions and they are trapped within the nerve cell.
- Potassium ions diffuse out of the membrane which restores the potential (repolarisation).
- The whole process takes about 2 ms.
- Then the potassium ions are pumped out, a process taking about 50 ms.

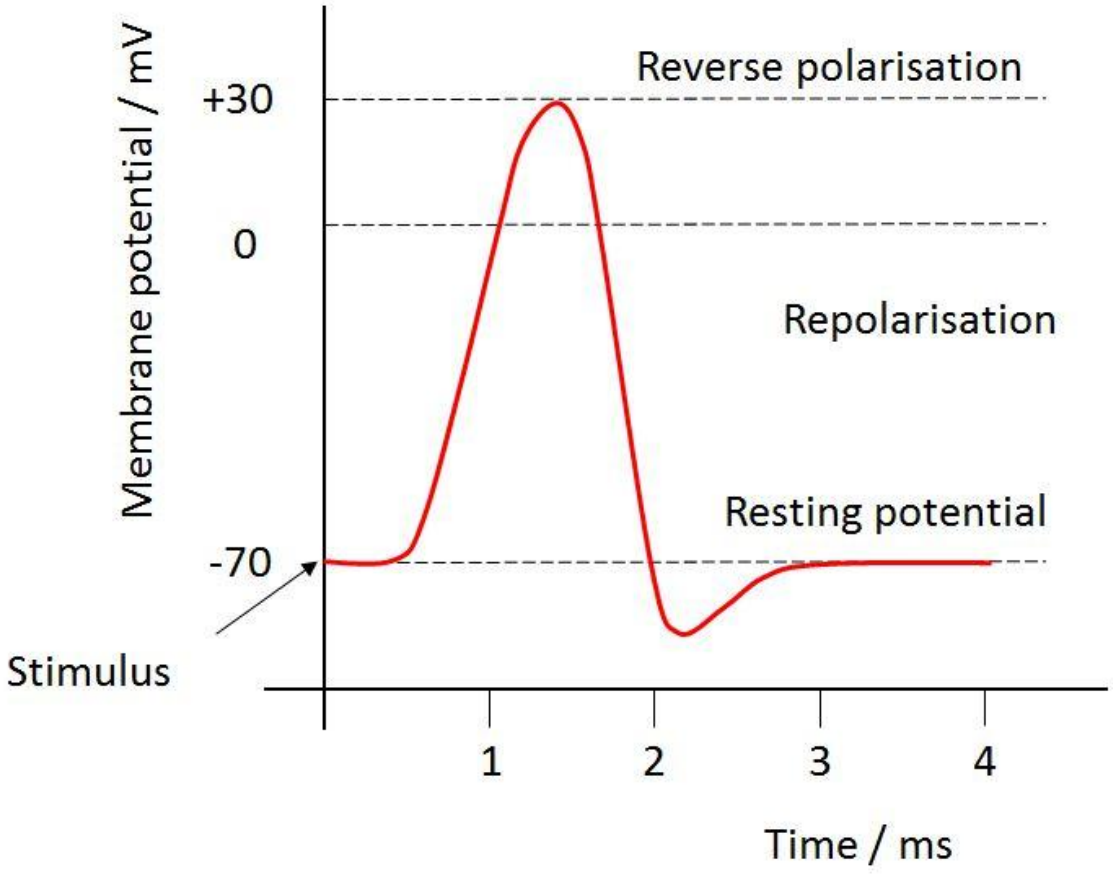


Figure 32 Action potential

The depolarisation is called an **action potential** (*Figure 32*).

14B.042 Nerve Cells

The action potential propagates along the nerve cell (**neurone** or **neuron** – either is correct) membrane at a rate of about 100 m s^{-1}. The nerve cell has a structure like this (*Figure 33*).

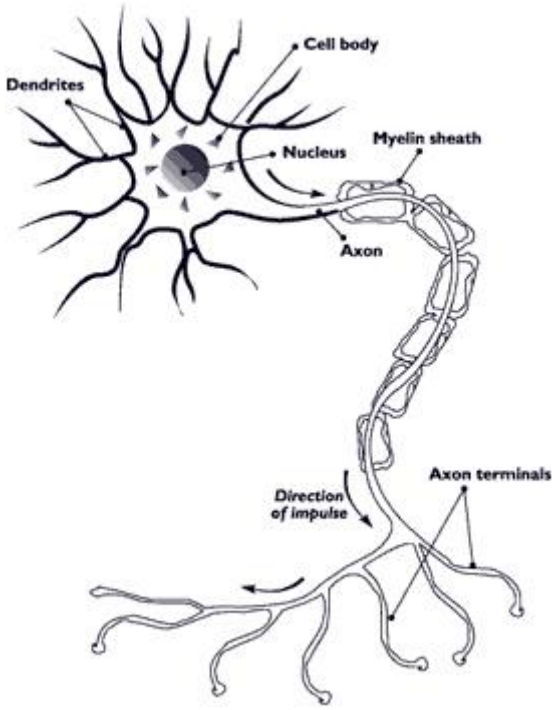


Figure 33 A neurone

The measurement of electrical activity in the body can be done in two ways:

- A concentric needle electrode which can be used to detect electrical activity in a very small area. This is invasive which means the body has to be penetrated. It can involve risk of infection.
- Surface electrodes which are placed on the body. A good electrical contact is made with a saline (sodium chloride) gel. This is non-invasive but cannot be used in very precise work.

Both types of electrode are connected to a high gain amplifier, and a pen recorder. The pen recorder makes a voltage time graph. Nowadays electrical activity can be logged into a computer.

Nerve cells make muscles contract, and the electrical behaviour of muscles can be followed in a similar way. The analysis of electrical signals is very useful for looking at:

- Activity of the heart.
- Brain activity, which can help the doctor to diagnose brain conditions such as epilepsy.

The Heart

The **heart** is a muscular bag that consists of four chambers (*Figure 34*):

- The **right atrium** which receives deoxygenated blood from the body
- The **right ventricle** that pumps the deoxygenated blood to the lungs. The pulmonary artery is the only artery that carries deoxygenated blood.
- The **left atrium** that receives oxygenated blood from the lungs. The pulmonary vein is the only vein that carries oxygenated blood.
- The **left ventricle** that pumps blood to the body through the **aorta**, a very large artery. We can think of it like a manifold that distributes blood to arteries that convey the blood to the whole body.

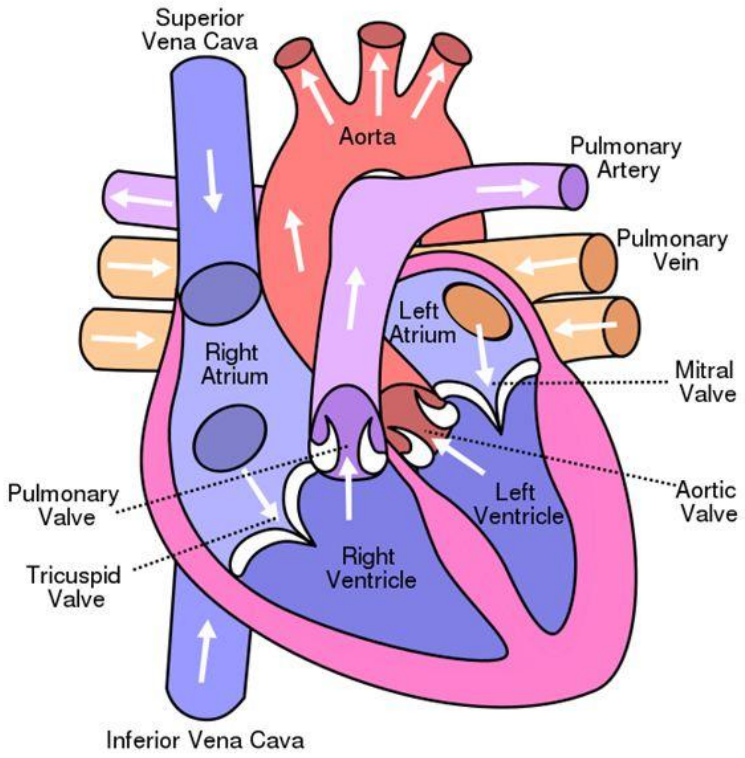


Figure 34 The heart (Image from Wikimedia Commons. Author not stated)

The heart is a **double pump**. The right hand side pumps blood at a low pressure to the lungs.

The left side of the pump is thicker and provides a higher pressure to get the blood around the rest of body (*Figure 35*).

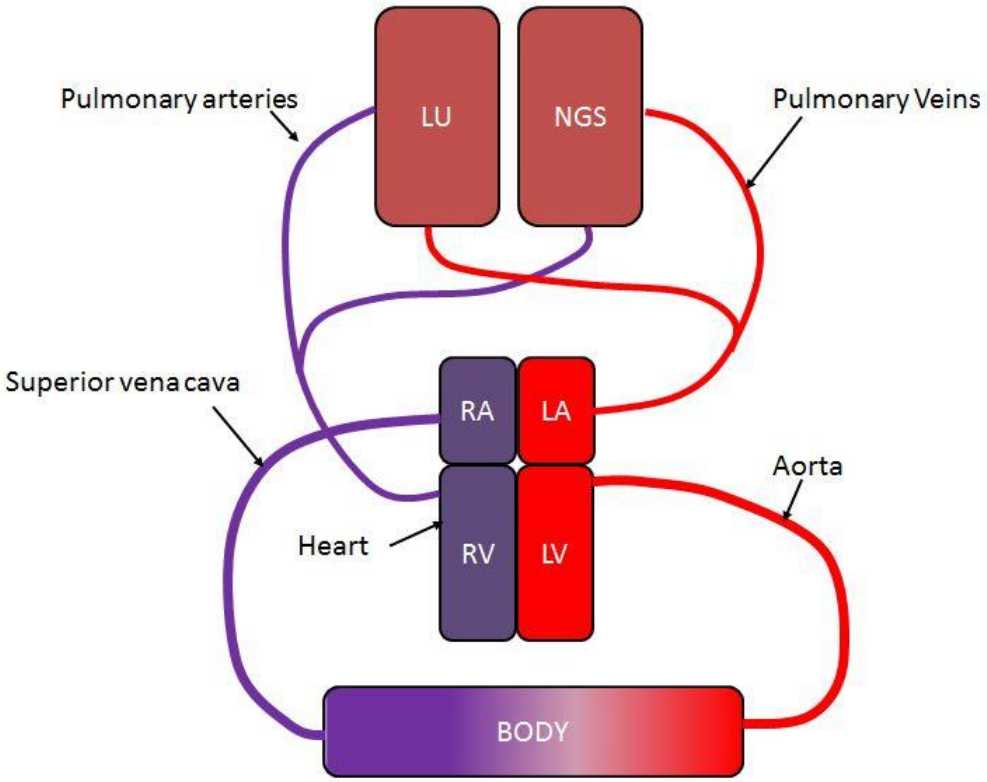


Figure 35 How the heart relates to the body

The heart is a muscular bag controlled by nerve cells. It has beat in a coordinated way, otherwise it will end up just twitching and not pumping blood at all, which is not very good for the health of the patient. The regular pumping action is controlled by a special set of cells called the **sino-atrial node** located on the right atrium. It produces an electrical stimulus about 70 times a minute, but higher in times of exercise.

- The atria contract, forcing blood into the ventricles through the valves.
- Then the ventricles contract; the valves from the atria are closed and the valves into the arteries are opened.

The action potential is shown (*Figure 36*):

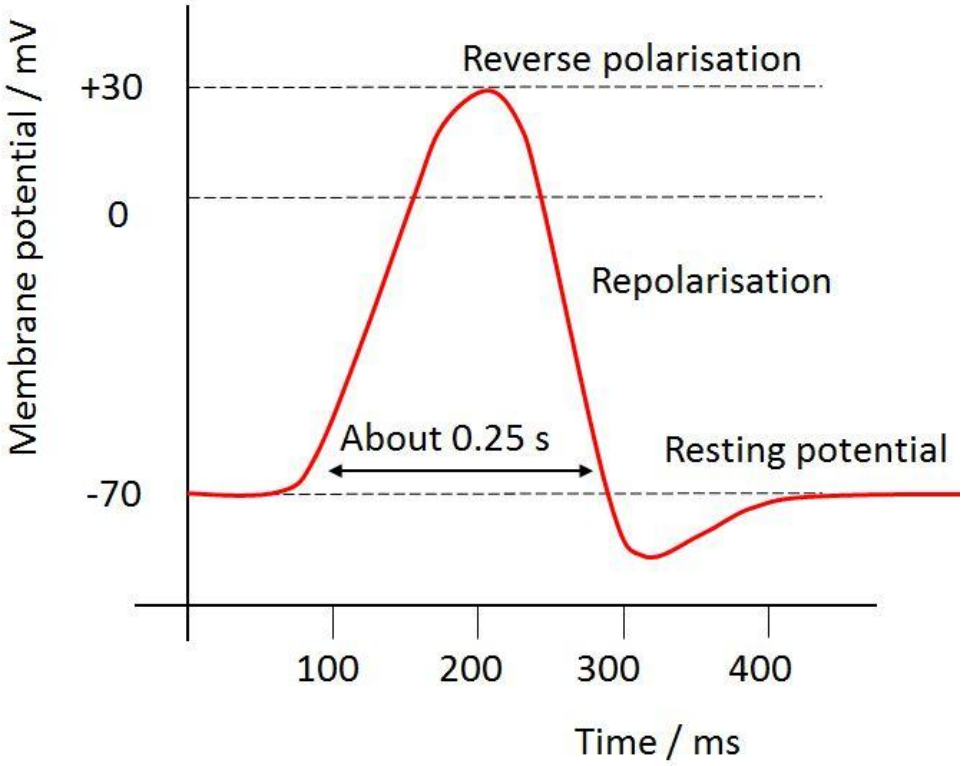


Figure 36 Action potential for the heart

Note that the action potential for the heart is rather slower than the action potential in a nerve cell.

14B.044 The Electrocardiogram

The **electrocardiogram** (ECG) allows doctors to look at the electrical behaviour of the heart. The conducting nature of body fluids transmits some of the electrical activity to the surface. The signals are much reduced in size and have amplitudes of about 1 mV. To get a good ECG the patient must be relaxed.

A typical ECG is shown (*Figure 37*):

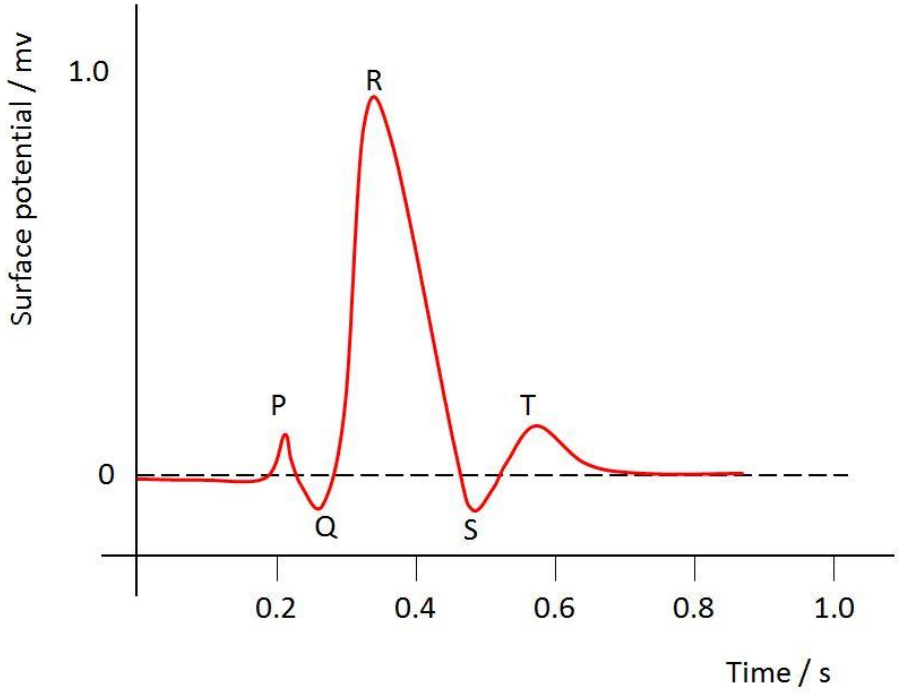


Figure 37 Electrocardiogram (ECG)

The important features are referred to by the letters shown on the graph:

- **P-wave** is due to the depolarisation and contraction of the atria.
- The **QRS-wave** is due to the depolarisation and contraction of the ventricles.
- The **T wave** is due to the re-polarisation and relaxation of the ventricles.

Note that the ECG is not the same as the heart's action potential. The whole process of a heart beat is set off using the action potential of the **sino-atrial node**. It is a group of cells on the right atrium of the heart. It is connected to the autonomic nervous system which detects increased demand for oxygen. Therefore, the heart rate goes up within three to five seconds of starting exercise. The brain is not involved.

To obtain an ECG, electrodes are placed on the arms and legs as well as the chest:

- Right arm.
- Left arm.
- Left leg.

The right leg is not used because it's furthest from the heart. The leads are named from a convention dating back to Einthoven who devised the first ECG machine in 1903. Nowadays the ECG is connected either to a chart recorder, or a computer. The general layout of the machine is shown (*Figure 38*).

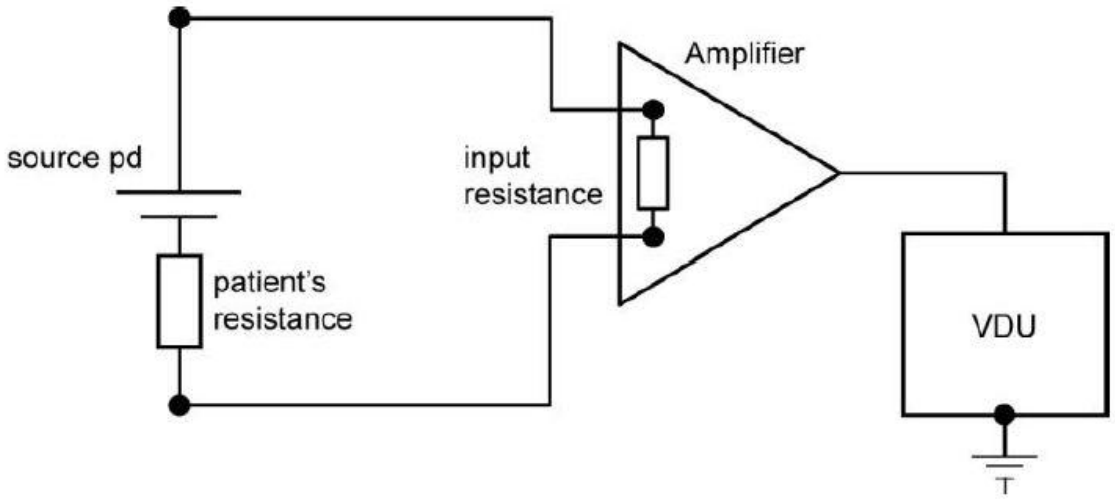


Figure 38 General layout of an electrocardiogram machine

The machine has these features:

- A **voltage gain** of about 1000. Compared with other voltage gains, this does not seem that high. Any higher and spurious signals from other muscular activity will be picked up. The frequency range is not very high, 0 - 20 Hz. The patient can pick up mains hum at 50 Hz, which would interfere with the signals from the heart.
- The **frequency response** must be even, so that the trace is not distorted.
- A **high input resistance** is essential, otherwise the p.d. will be reduced because of the body resistance and a certain amount of capacitance. Contact resistance is reduced by using conducting gel between the skin and the electrodes. The circuit behaves rather like a potential divider.
- High **signal to noise ratio**. Random signals can be given off in electronic circuits. It can be heard as "white noise" in an audio amplifier. These signals could mask small changes.

The ECG can be used to diagnose problems with the heart which include:

- **arrythmia**, an irregular pumping pattern which is quite common in young people, increased by exercise.
- blockage of part of the blood supply for the heart. This can lead to the heart muscle getting tired, or in extreme cases death of the heart muscle, **myocardial infarction** (heart attack).
- **fibrillation** in which there is no co-ordinated pumping activity, which will lead to death if not treated very quickly.

Defibrillation is a dramatic intervention. Paddles are placed over the heart, and a brief massive electric shock is given, causing a major contraction of all the heart muscles. This is often gets the heart beating in a regular way again.

The role of the sino-atrial node can be taken over by an **artificial pacemaker,** which produces about 70 beats a minute. This can give a heart patient a good quality of life.

When I was a young student, we did a practical in the physiology lab that involved hooking ourselves up to an ECG machine. My trace was sufficiently odd to get the lecturers to consult the professor (an expert cardiologist). A dead student in the lab would not give the university good publicity. However, my heart did not pack in. The cardiologist suggested that it lies on its side, rather than upright. It has since reliably driven me across many kilometres of rowing races and fell runs. And I don't intend to drop dead yet, despite the heartfelt wish of one or two of my students...

I wrote the last sentences a few years ago. I now have some health issues.

Questions

Tutorial 14B.04

14B.04.1

Why must the blood leaving the heart for the lungs be at a much lower pressure than the blood leaving the heart through the aorta?

14B.04.2

W hen having an electrocardiogram, why does the patient need to be relaxed?

14B.04.3

The operator using a defibrillator often shouts "stand clear" before operating the machine. Why?

2. Imaging Techniques

Tutorial 14B.05 Ultrasound Imaging

AQA Syllabus

Contents

14B.051 Initial Diagnosis

For most people the doctor can diagnose an illness by:

- Listening to the patient describing symptoms.
- Listening to body sounds through a stethoscope.
- Feeling the patient's body.
- Taking the patient's temperature.
- Looking down the ears and throat.

The doctor may need to carry out further tests by taking samples of urine, or blood, which are analysed in the hospital laboratory.

For some patients more specialised diagnostic techniques are needed, which are what we are going to look at here. They are usually carried out in a hospital and require specialist personnel to operate the machines and to interpret the results. Before these techniques were invented, the only way to diagnose problems was to feel the body. An exploratory operation, a **laparoscopy**, could be carried out, but this **invasive** procedure carried serious risks. Such operations can still be carried out as a last resort.

These techniques we will look at are **non-invasive**. They enable the doctor to see under the skin without having to open up the body. An **invasive** procedure requires cutting the

skin. In this tutorial and Tutorial 14B.06 we will look at diagnostic techniques that do **not require ionising radiation**. In Tutorials 14B.07 and 14B.08 we will look at those techniques that use ionising radiation such as **X-rays** and **gamma rays**.

14B.052 Ultrasound Imaging

Ultrasound is any sound that is higher than the upper limit of human hearing.

Bats and dolphins use frequencies in the range of 30 - 100 kHz for echo location and in the Second World War experiments in ultrasound were tried out to detect submarines. Ultrasound imaging is a proven method of investigating objects internally without causing damage. It is widely used in medicine because it is **non-invasive**.

Higher frequency waves give greater resolution, so make clearer images.

14B.053 Generation and Detection of Ultrasound

The ultrasound probe or **transducer** is used to generate and detect the ultrasound waves. The most common method is by use of a **piezoelectric transducer**. If you squeeze or stretch a crystal of quartz (easier than might be thought) a voltage is induced. It can be high enough for a spark to jump. Gas lighters use the effect. Conversely, if you apply a voltage to a piezoelectric material, you can make it change shape. If the voltage is alternating, the crystal will vibrate. Maximum energy transfer occurs when the crystal is **in resonance**.

The material, usually **lead zirconate titanate** (PZT), an artificial ceramic, has a thickness of half a wavelength of the ultrasound wave.

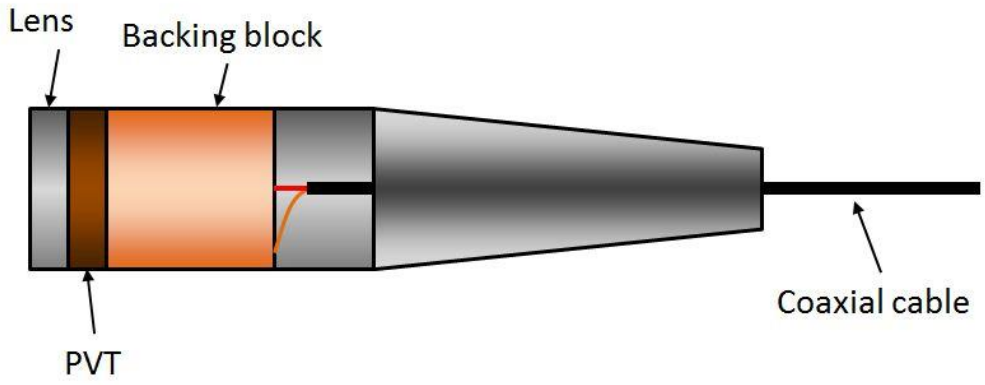


Figure 39 An ultrasound probe

The lens protects the PZT slice and acts to converge the beam slightly. The beam consists of short pulses of frequencies of several megahertz. The vibrations are damped by the backing block which is made of epoxy resin. The whole is contained in a metal case which protects the probe mechanically and electrically.

The ultrasound probe sends out **pulses**, rather than a continuous beam. This allows the reflected waves to be detected. The same crystal detects the reflected waves. The frequency of the pulses used in medical ultrasound scanners are between 2 to 15 MHz. The probe emits trains of waves (pulses) that last about 15 ms. Then no pulses are sent out for about 0.5 ms, so that the machine can detect the echoes. The idea is shown below (*Figure 40*).

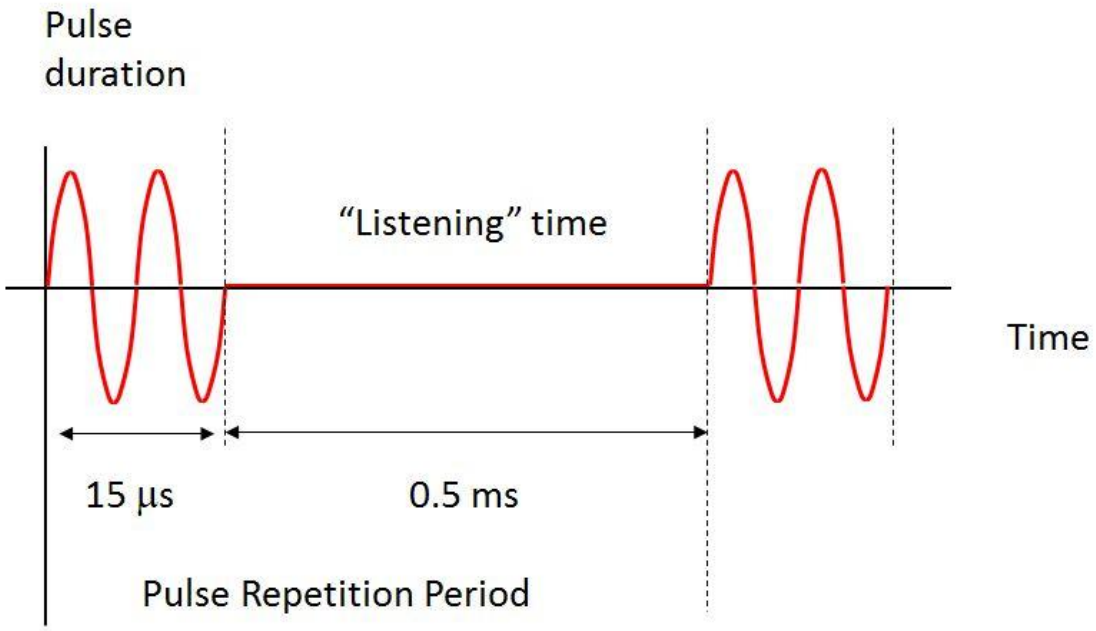


Figure 40 Ultrasound pulses

Worked Example The frequency of an ultrasound wave is 2.5 MHz. Each pulse duration is 15 ms. How many waves are in each train of waves?
Answer Each wave lasts $1 \div 2.5 \times 10^{6}$ Hz = 4.0×10^{-7} s In 15 ms, there are 15×10^{-6} s ÷ 4.0×10^{-7} s = **37.5 waves**

The **Pulse Repetition Period** (PRP) is the sum of the pulse and the "listening time". In this case (*Figure 40*) it is:

$$PRP = 15 \times 10^{-6}\ s + 0.50 \times 10^{-3}\ s = 5.15 \times 10^{-4}\ s = 515\ ms$$

Therefore, the probe would send out 1941 such pulse repetitions every second.

The ratio of the **pulse duration** to the **pulse repetition period** is often called the **duty factor** (DF). It given as a percentage. In this case:

$$DF = (15 \times 10^{-6}\ s \div 5.15 \times 10^{-4}\ s) \times 100\ \% = 2.9\ \%$$

14B.054 Ultrasound in the Body

When ultrasound passes into the body:

- it is a longitudinal wave travelling in a material.
- it is reflected at boundaries between different tissues.
- it is absorbed by tissues.

The extent to which the ultrasound is absorbed or reflected gives information about the structures below.

The table below gives some ultrasound properties of some body tissues:

Material	***Density*** **(kg m^{-3})**	***Velocity*** **(m s^{-1})**	***Acoustic Impedance*** **($Z = \rho c$)** **(kg m^{-2} s^{-1})**
Air	1.3	330	429
Water	1000	1500	1.50×10^6
Blood	1060	1570	1.59×10^6
Brain	1025	1540	1.58×10^6
Fat	925	1450	1.38×10^6
Muscle	1075	1590	1.70×10^6
Bones (varies)	1908	4080	7.78×10^6
PZT Transducer	7650	3791	29.0×10^6
Quartz Transducer	2650	5736	15.2×10^6

The **acoustic impedance** is the product of the **density** and **speed** of sound in the material.

$$Z = \rho c$$

.....*Equation 9*

Z - acoustic impedance (kg m^{-2} s^{-1}); ρ - density (kg m^{-3}); c - speed of sound (m s^{-1})

Remember that the waves emitted from the ultrasound probe are **reflected** and picked up by the probe. Therefore, the time taken for the waves to get through a layer is **half** the time measured by the probe (*Figure 41*).

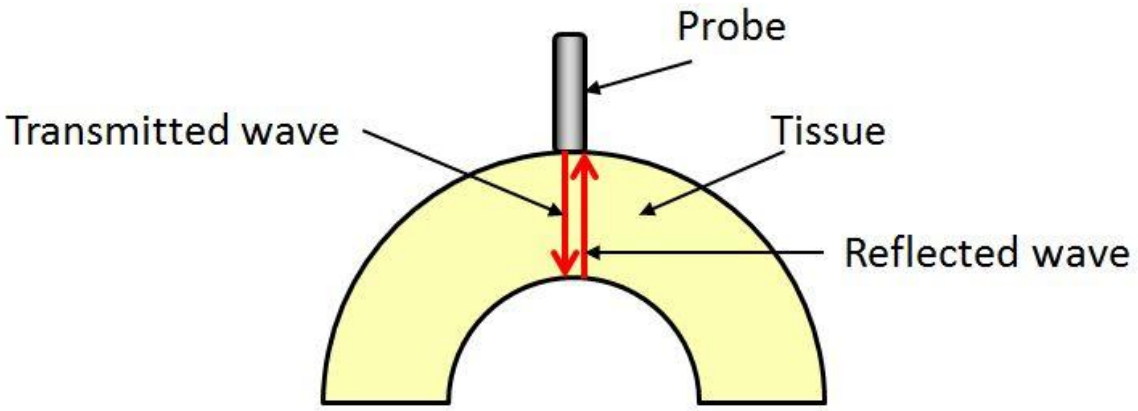


Figure 41 Ultrasound is reflected at a boundary

If the probe is placed straight on to the skin, almost all the energy will be reflected. So, the probe has to have a **coupling medium** between it and the skin. This is a gel or an oil. If there is gas anywhere, it can cause big problems for imaging.

When the waves reach a boundary, a small amount, about 1 % gets reflected. However, if the difference in acoustic impedance between two tissues is large, a higher proportion of the ultrasound waves are reflected. For example, the boundary between lung tissue and the air in the lungs leads to a 99.9% reflection, making it impossible to view structure behind the lungs.

Probes are expensive, about £4000 (€5000) each.

14B.055 Imaging in the Body

The ultrasound passes through the tissue and is **reflected** at various boundaries as shown (*Figure 42*):

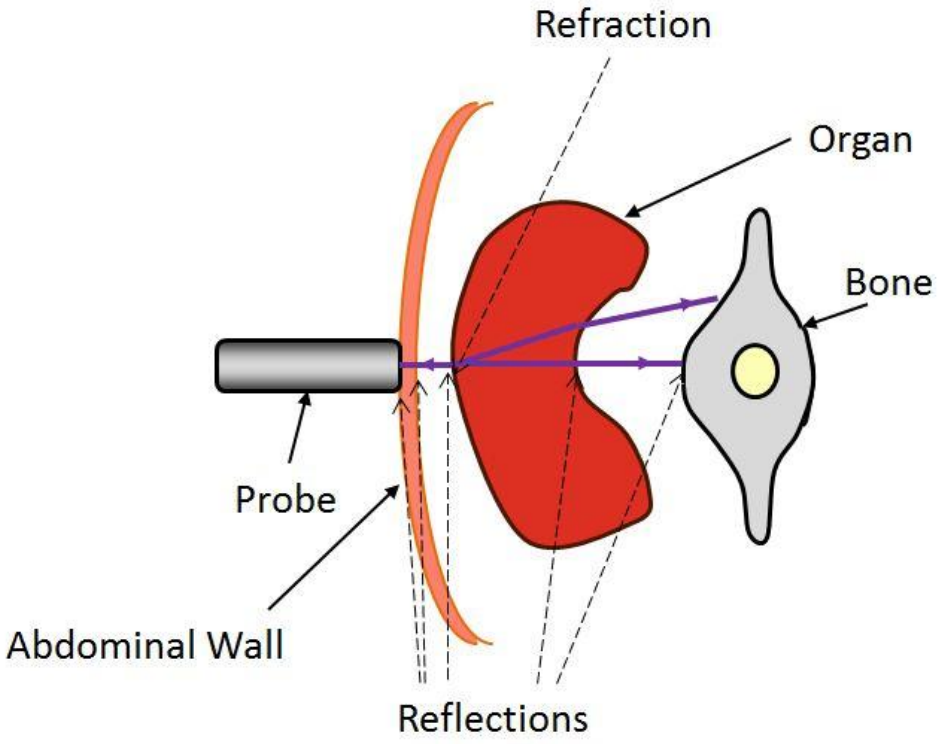


Figure 42 Passage of ultrasound through the organs of the body

There will also be **refraction**, as the wave speed changes as it passes across boundaries. For example, the speed of sound through water is 1500 m s^{-1} compared with 1450 m s^{-1} through fat.

The **resolution** of the beam means the smallest distance that can be discriminated in the image. The higher the frequency, the better the resolution. However, as the beam passes through, the sound waves get scattered and absorbed by the molecules. This **attenuation** is more marked with higher frequency. Therefore, a compromise has to be made. The optimum frequency for imaging the brain and abdomen is roughly 1 - 3 MHz.

- **Axial resolution** is the resolution in the direction of the beam. It can be improved by **making the pulses short**. While the transducer is producing pulses, it cannot detect echoes. Therefore it makes sense to make the pulses as short as possible.
- **Lateral resolution** is determined by the **beam width**. If two structures are within one beam, they cannot be discriminated.

- Resolution is limited by **diffraction** effects. Just like in light, objects less than 1 wavelength apart cannot be resolved, the same is true of sound. 1 MHz waves can discriminate structures that are 1.5 mm apart.

When the return pulses are received, the transducer turns them into electrical signals to be stored:

- using video tape.
- on a storage CRO.
- as digital data for analysis by a computer.

Many dedicated ultrasound scanners can give a pictorial output. For example, an ultrasound scan can be made for the bladder, to tell how much urine there is in the bladder. This tells the nurse whether a patient's bladder has fully emptied.

14B.056 Scans

There are a number of ways in which ultrasound can be used. We will consider:

- **A-scan**, which is the amplitude modulated display
- **B-scan**, which is a brightness modulated display.

The two are compared in the diagram below (*Figure 43*):

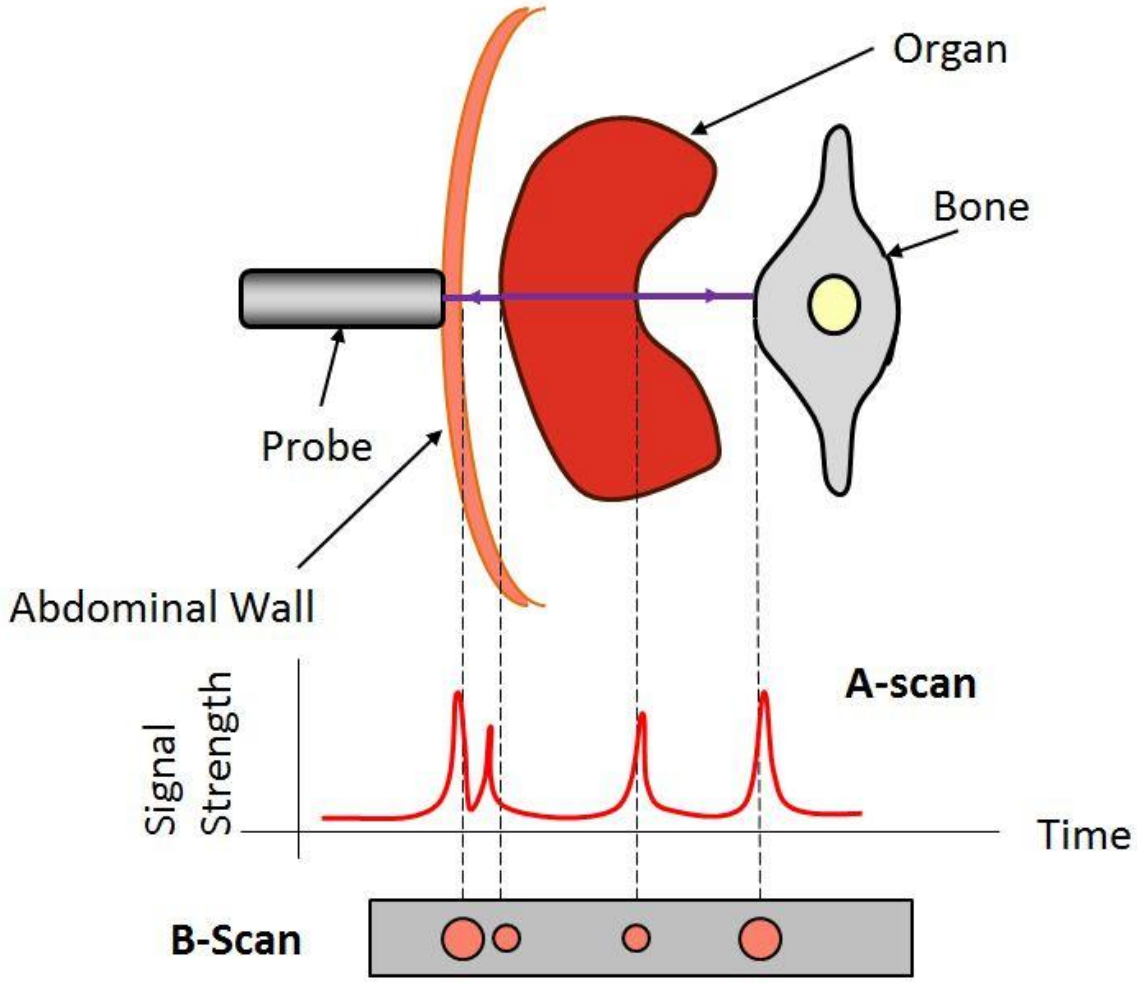


Figure 43 A- and B-scans in ultrasound imaging

A-scans are used where the anatomy of a section is well known, and a precise depth measurement is needed. One example is where the position of the midline of the brain is needed. Any delay could indicate the presence of a tumour or a fluid filled space.

The B-scan is the basis of two-dimensional scanning. The transducer is moved about to view the body from a variety of angles. The probe can be moved in a line, a **linear scan**, (*Figure 44*) or rotated from a particular position, **sector scan** (*Figure 45*).

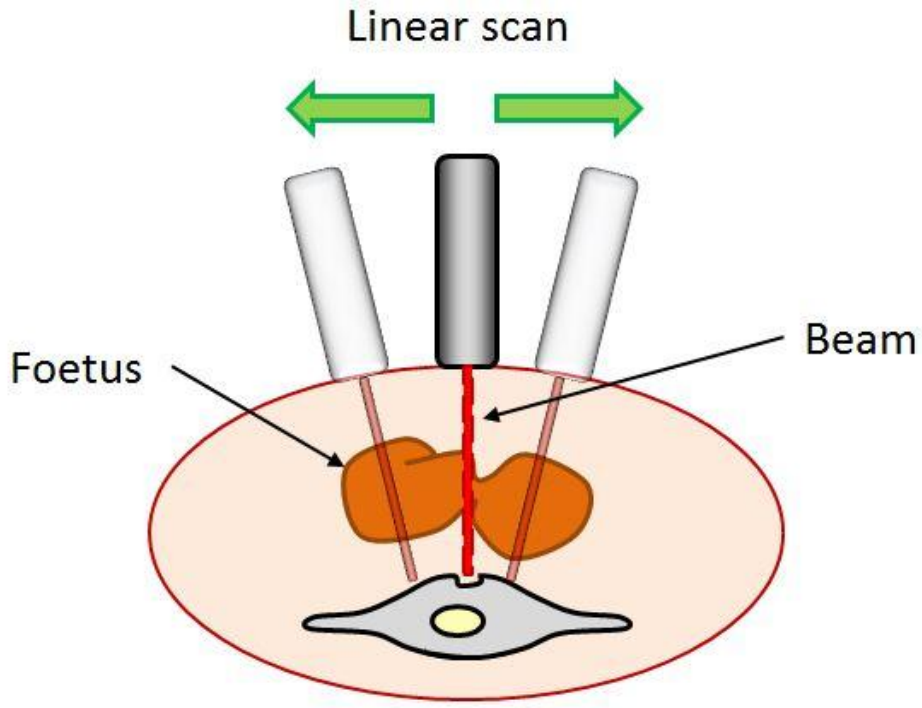


Figure 44 A linear scan

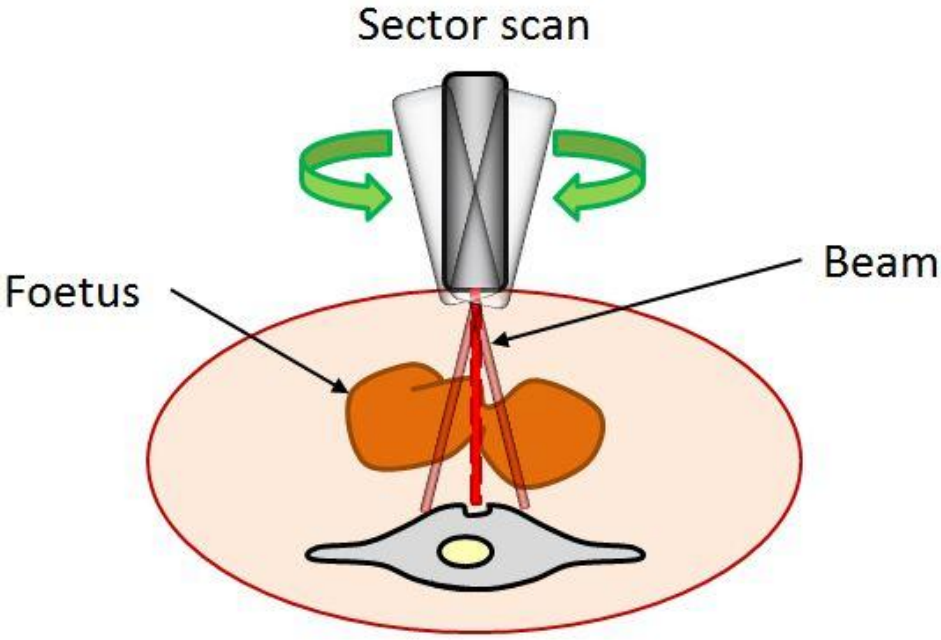


Figure 45 A sector scan

The two movements can be combined to give a **compound scan**. It requires considerable skill and a good knowledge of anatomy for the **sonographer** to get a decent image and to interpret it. As you can see, it takes some interpreting. *Figure 46* shows a foetus during pregnancy, a very common use for ultrasound. The head is on the bottom right, while the legs are to the top left.

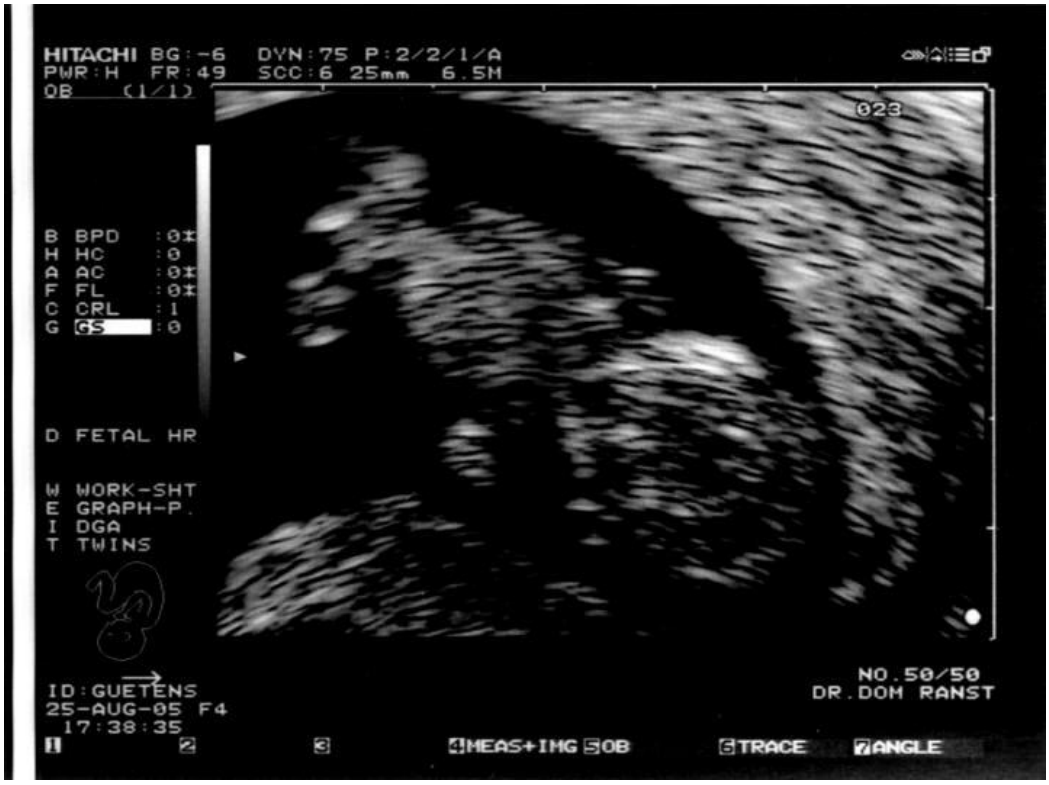


Figure 46 A typical ultrasound scan of a foetus (Image from Wikimedia Commons (Melimama))

Ultrasound is used in other investigations such as detections of cysts, abscesses, and tumours.

Real time B-scans use a linear array of up to 100 transducers to get a cross section of the body. Moving images are possible.

Ultrasound can be used with the **Doppler** effect to watch the movement of blood through the blood vessels. This might be used to check good blood flow in the feet of a diabetes patient.

14B.057 Advantages and Disadvantages of Ultrasound Scanning

Ultrasound is generally a very safe diagnostic technique:

- There are no known hazards with low frequency (low energy) beams.
- It is non-invasive.
- There is no discomfort apart from a cold probe!
- More effective than X-ray techniques in producing images of soft tissue.
- The equipment is relatively inexpensive, can be moved about very easily, and does not need a specialist room.
- There are no hazards for the operator.

However:

- The sonographer has to be skilled at operating the probe and its associated equipment to get a decent image.
- The image needs skilful interpretation.
- Attenuation can reduce the resolution of the image.
- Bone absorbs ultrasound so that brain images are hard to get.
- Gas-soft tissue interfaces reflect 99.9% of the incident energy. Images of tissues on the far side of lungs are impossible to get.

High energy ultrasound waves can be used for therapeutic purposes. Low intensity ultrasound can be used in healing wounds and relieving discomfort and pain in conditions like arthritis. High intensity beams can shatter kidney stones. Ultrasound treatment has to be done with care because:

- the temperature in the tissues can rise.
- the pressure changes can rupture cells.
- bone is a strong absorber of ultrasound.

While this would not cause many problems for an adult, it must be avoided where there is a growing foetus.

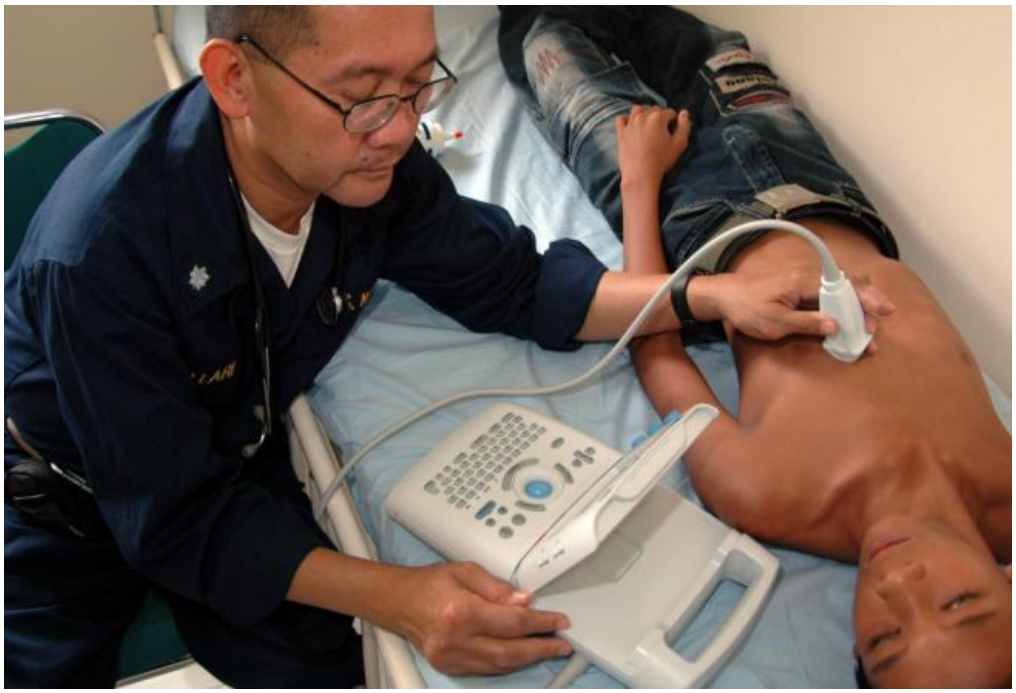


Figure 47 A sonographer takes an ultrasound scan on a patient (Photo by Joseph Caballero, Wikimedia Commons)

14B.058 Acoustic Impedance

We have seen before how acoustic impedance is defined as:

the product of the density and speed of sound in the material.

$$Z = \rho c$$

..... *Equation 10*

Z - acoustic impedance (kg m^{-2} s^{-1}); ρ - density (kg m^{-3}); c - speed of sound (m s^{-1})

The unit for acoustic impedance is sometimes called the **Rayl** (named after John William Strutt (1842 - 1919), 3rd Baron Rayleigh).

When ultrasonic sound waves are applied to any system consisting of any material, the materials will present some kind of opposition to the passing of the sound waves. Therefore, the wave energy is lost. The intensity of the reflected waves received at the probe will always be less than the intensity of those that are transmitted. The extent to which the energy is lost in a material is called the **acoustic impedance**.

When wave energy strikes the interface between two different materials, a certain amount of the energy is **reflected**. This is why you can see your reflection in the glass of a window when it's dark outside. We saw that when we did refraction with light. There was always a weak reflected ray, which we tended to ignore. The same is true with sound waves, which is why you hear echoes from walls. (You may remember doing an experiment to measure the speed of sound by standing 30 metres from the wall of the gym, banging two bits of wood together, and banging them again when you heard the echo. You timed 10 bangs, etc. I hated doing that experiment, as I could never get the rhythm...)

14B.059 Refraction in Ultrasound

When ultrasound is emitted by the probe, the probe acts as a point source, with the waves propagating as a cone. The idea is shown below (*Figure 48*):

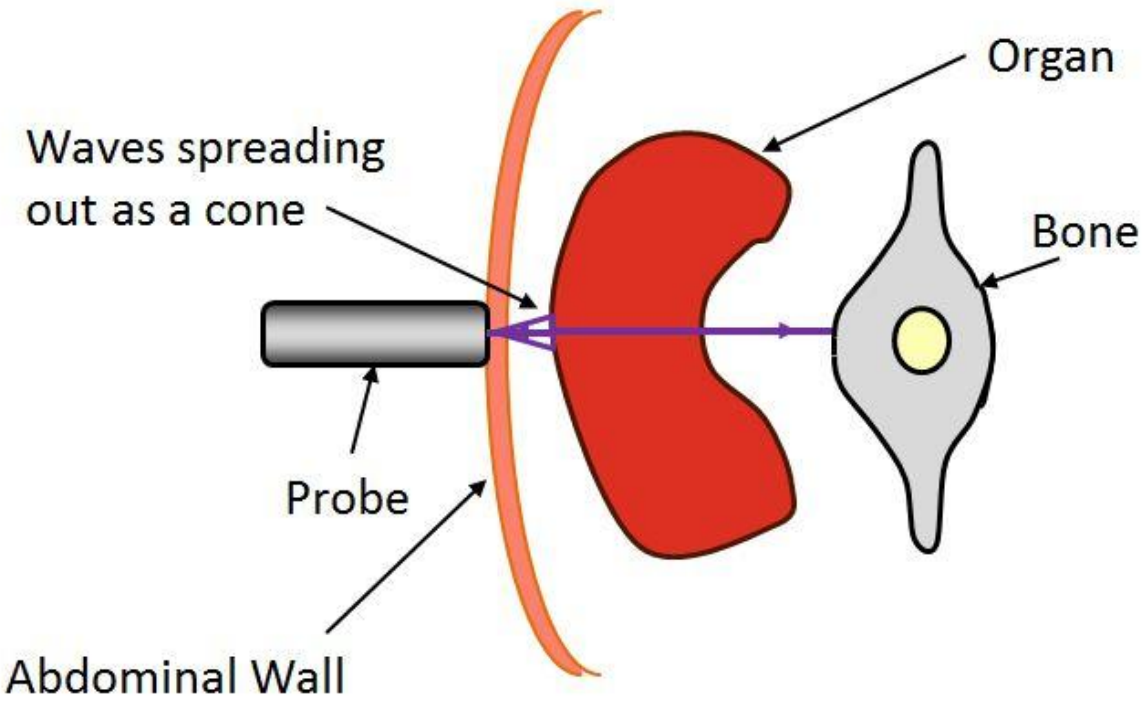


Figure 48 Ultrasound waves propagate as a cone

As the waves pass across the boundary between the abdominal cavity and the organ, they pass from water of acoustic impedance 1.50×10^6 kg m^{-2} s^{-1} to a tissue of a different acoustic impedance. For the liver, $Z = 1.69 \times 10^6$ kg m^{-2} s^{-1}.

Before you carry on, you may wish to revise Topic7 – Waves. From Topic 7, we know that:

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

..... *Equation 11*

Do NOT use the refractive indices that you might know for optical refraction. We are using sound waves, not light. Most tissues in the body are **opaque** to light.

It is quite possible that you might get asked a question like 14B.05.10 in the exam. Refraction will spread the waves out as they pass boundaries (if the speed of the waves is higher in the second material). This will have the effect of reducing further the intensity of the waves. It could also affect the results if there are several probes being used to pick up transmitted waves.

14B.0510 Reflection of Ultrasound

We have discussed how the waves can be scattered by reflection and refraction. However, for a single probe ultrasound scan, that isn't really an issue. The sonographer is only interested in the waves that are transmitted and received by the probe. The received waves are those that are reflected by the tissue or organ under study. The acoustic impedance, Z, is the important quantity in this case.

The fraction of the energy reflected is sometimes called the **reflection fraction**, or the **reflection coefficient**. It is expressed as a fraction or a percentage. It is always less than 1, because some energy is lost by reflection, refraction, and absorption. If you get an answer greater than 1, you have done something wrong. The reflection fraction is sometimes given the code Γ. (The symbol that looks like a gallows is Gamma, a Greek upper-case letter 'G'). It is related to the intensity in the equation:

$$\Gamma = \frac{I_r}{I_i}$$

..... *Equation 12*

Γ - reflection fraction; I_r - Intensity of the reflected wave (W m^{-2}); I_i - Intensity of incident wave (W m^{-2})

We can extend this further by relating the reflection fraction to the acoustic impedance with this relationship:

$$\frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

..... *Equation 13*

Z_1 - acoustic impedance of material 1 (kg m^{-2} s^{-1}); Z_2 - acoustic impedance of material 2 (kg m^{-2} s^{-1})

The way to remember this is write:

Reflection fraction = (difference in acoustic impedances ÷ sum of the acoustic impedances)^2

It doesn't matter if the difference yields a negative number, because the whole thing is squared.

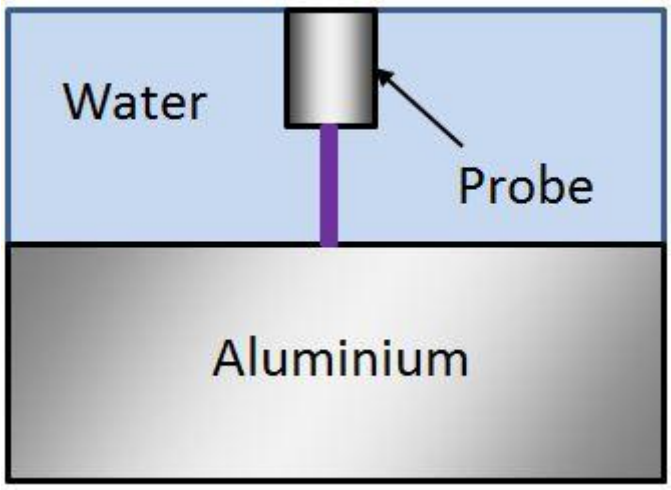
Worked example

An ultrasound probe has a power of 3.0 W. It forms a circular beam of diameter 2.5 cm and can be modelled as having parallel sides.

(a) Work out the intensity of the of the beam.

(b) The density of aluminium is 2700 kg m^{-3} and sound travels through aluminium at a speed of 6320 m s^{-1}. Calculate the acoustic impedance of aluminium.

(c) An ultrasound probe is placed near a block of aluminium as shown:



Calculate the reflection fraction from the aluminium.

(d) Calculate the intensity of the reflected wave, assuming that the transmitted wave has not lost any of its energy in the water.

Answer

(a)

$$I = P \div A$$

Area $= \pi D^2/4$ = (π × (2.5 × 10^{-2} m)$^{2)}$ ÷ 4 = 4.91 × 10^{-4} m^2

I = 3.0 W ÷ 4.91 × 10^{-4} m^2 = **6112 W m^{-2}**

(b)

$Z = \rho c$ = 2700 kg m^{-3} × 6320 m s^{-1} = **17.0 × 10^6 kg m^{-2} s^{-1}**.

(c)

Z for water = 1.50 × 10^6 kg m^{-2} s^{-1}. Z for aluminium = 17.0 × 10^6 kg m^{-2} s^{-1}.

Formula:

$$\Gamma = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

Γ = ((17.0 × 10^6 kg m^{-2} s^{-1} - 1.50 × 10^6 kg m^{-2} s^{-1}) ÷ (17.0 × 10^6 kg m^{-2} s^{-1} + 1.50 × 10^6 kg m^{-2} s^{-1}))^2

= ((15.5 × 10^6 kg m^{-2} s^{-1}) ÷ (18.5 × 10^6 kg m^{-2} s^{-1}))^2

Γ = 0.838^2 = 0.702 = **70.2 %** (Remember that you have to square it.)

(d) Formula:

$$\Gamma = \frac{I_r}{I_i}$$

$I_r = \Gamma I_i$ = 0.702 × 6112 W m^{-2} = 4290 W m^{-2} = **4300 W m^{-2}** to 2 s.f.

Note that this example uses a non-biological material. The principles of ultrasound are equally valid in engineering. Ultrasound probes are widely used in the engineering industry to check for flaws in casting, etc.

Remember to **square** the term [(Z_2 - Z_1) ÷ (Z_2 + Z_1)]

From this answer to 14B.05.10, we can see where the acoustic impedances are similar, the fraction of energy reflected is small. They will not show up well against the muscle. Ultrasound is useless to look things going on in the lungs, because they are full of air. The next question, 14B.05.11 shows why.

Note that the frequency of the ultrasound is NOT affected by reflection, absorption, refraction, or attenuation.

Investigations of the lungs require different techniques for investigation, for example, endoscopy.

14B.0511 Blood Flow (OCR, Eduqas, and Welsh)

Ultrasound can be used to check the flow of blood into an organ. If there is insufficient flow of blood, the organ might not work properly. It could be seriously damaged which can cause permanent damage to the patient.

The blood flow can be measured using a probe held at an angle as shown below (*Figure 49*):

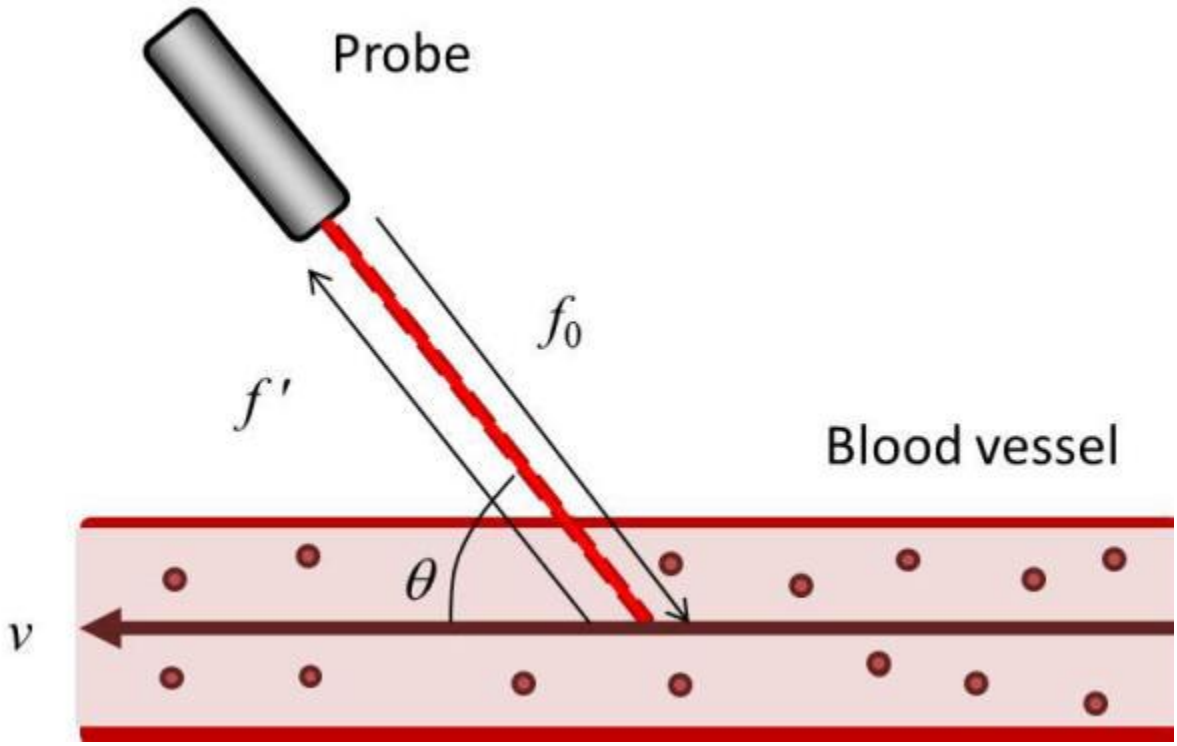


Figure 49 Measuring blood flow using an ultrasound probe

The ultrasound waves are reflected from the red blood cells. Since the red blood cells are moving from right to left (towards from the probe), the reflected frequency f' (f-prime) is higher than the transmitted frequency f_0, due to the **Doppler Effect**. (If the probe were in the other direction, the reflected frequency would be lower than the transmitted frequency.)

The factor we are interested in is the difference in the frequency, given the physics code Δf. This is shown in the equation below:

$$\Delta f = f_0 - f'$$

..... *Equation 14*

The Doppler Effect equation tells us that the ratio of the difference in the frequency to the original frequency is the same as ratio of the speed of the moving object to the speed of the wave. This is shown here:

$$\frac{\Delta f}{f_0} = \frac{v}{c}$$

..... *Equation 15*

[Δf - change in frequency (Hz); f_0 - original frequency (Hz);

v - speed of blood flow (m s^{-1}); c - speed of ultrasound (m s^{-1})]

Ideally, we would place the probe along the axis of the blood flow. However, that is not possible, so we have to have the probe at an angle, θ.

To get the speed of the blood flow we use the equation:

$$\frac{\Delta f}{f_0} = \frac{2v\cos\theta}{c}$$

..... *Equation 16*

The v term is doubled, because the waves are being transmitted from a fixed point, rather than a moving object.

Questions

Tutorial 14B.05

14B.05.1

What is the upper limit of human hearing?

14B.05.2

What are the conditions needed for resonance?

14B.05.3

A sound wave has a frequency of 5.0 MHz and travels at 1500 m s^{-1}. What is the wavelength?

14B.05.4

Use the table on page 52 for data on different materials.

What is the thickness of a slice of PZT material if its fundamental resonant frequency is 1.5 MHz?

14B.05.5

Use the table on page 52 for data on different materials.

The time delay for a pulse going through fat is 0.133 ms. How deep is the fat?

14B.05.6

Write down two advantages of using ultrasound as a diagnostic tool and two disadvantages.

14B.05.7

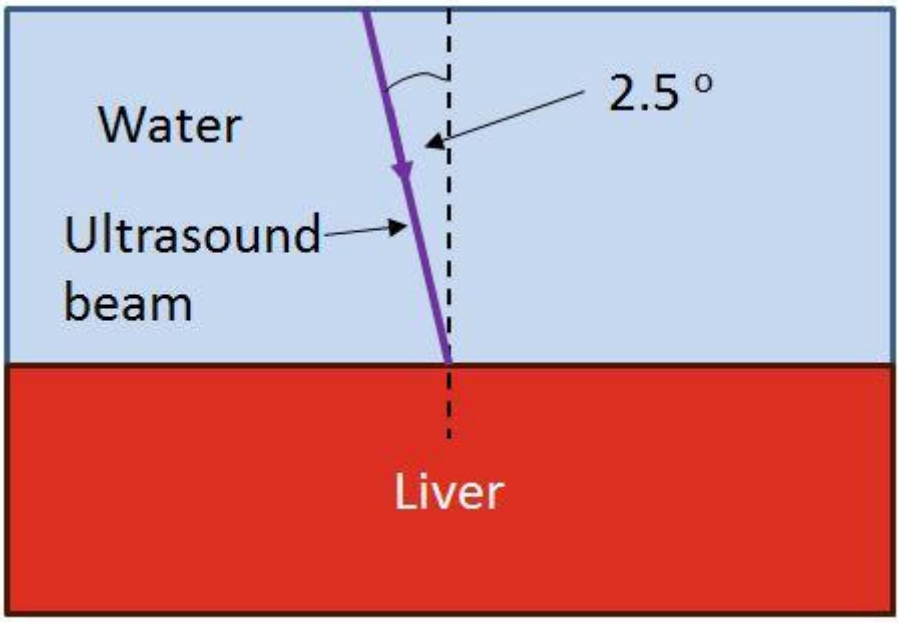
Show that the acoustic impedance of air is about 400 kg m^{-2} s^{-1}, assuming the density of air is 1.2 kg m^{-3} and the speed of sound in air is about 340 m s^{-1}.

14B.05.8

Calculate the speed of sound in the liver. The density of the tissue for liver is 1.06×10^3 kg m^{-3}. Give your answer to an appropriate number of significant figures.

14B.05.9

An ultrasound beam is incident on the surface of a liver at an angle of 2.0 ° as shown:



The speed of sound in water is 1500 m s^{-1}. Use your answer to Question 8 to calculate the angle of refraction.

What is the path of the beam after the refraction? Explain your answer.

14B.05.10

In the human body, the kidneys are in close proximity to the muscles of the back. An ultrasound probe sends waves to the kidney. By the time the waves reach the junction between the kidney and the muscles, they have an intensity of 260 W m^{-2}.

Using the data below, calculate:

(a) the acoustic impedance for the kidney tissue, and the muscle tissue.

(b) the intensity of the reflected wave.

Density of kidney tissue = 1050 kg m^{-3}; Speed of sound in kidney tissue = 1570 m s^{-1}

Density of muscle tissue = 1065 kg m^{-3}; Speed of sound in muscle tissue = 1590 m s^{-1}

14B.05.11

An ultrasound probe is being used to diagnose a problem with a patient's lung.

Lung tissue has an acoustic impedance of 2.60×10^5 kg m^{-2} s^{-1}.

Air has an acoustic impedance of 429 kg m^{-2} s^{-1}.

(a) Calculate the percentage of the intensity that is reflected.

(b) Comment on whether an ultrasound scan would be able to pick up any problems within the lungs.

14B.05.12

An ultrasound probe transmits at a frequency of 2.0000 MHz. It is set up an angle of 20 ° to a blood vessel to check the speed of flow. The received signal at the probe is found to be 2.0055 MHz.

(a) Calculate the speed of the blood, assuming the waves travel through the body tissue is about 1600 m s^{-1}.

(b) What would be the effect if the probe were held at 90 ° to the blood vessel.

Tutorial 14B.06 Endoscopy and MRI Scanning	
AQA Syllabus	
Contents	
14B.061 Endoscopy	14B.062 Image formation
14B.063 The Endoscope	14B.064 Lasers
14B.065 Safety Issues with Lasers	14B.066 Magnetic Resonance Scanner
14B.067 Benefits and Drawbacks	14B.068 Larmor Frequency *Welsh and Eduqas only*

14B.061 Endoscopy

Endoscopy is non-invasive in that the skin does not need to be broken. However, the endoscope has to be inserted through the nose, mouth, urinary tract, or anus. None of these are pleasant experiences for the patient. (I know; I have experienced such a procedure for myself.)

Endoscopes are based on **Optical fibres** are basically long thin glass rods which are surprisingly flexible. They conduct light by **total internal reflection**.

The light is guided down a **core** surrounded by glass **cladding** of slightly lower refractive index. This is to prevent the loss of light if the core were to come into contact with material of a higher refractive index.

When dealing with optic fibres we use the **Snell's Law Equation**:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

..... *Equation 17*

You may wish to revise this in Topic 7

Here is how light is conducted in a glass fibre (*Figure 50*):

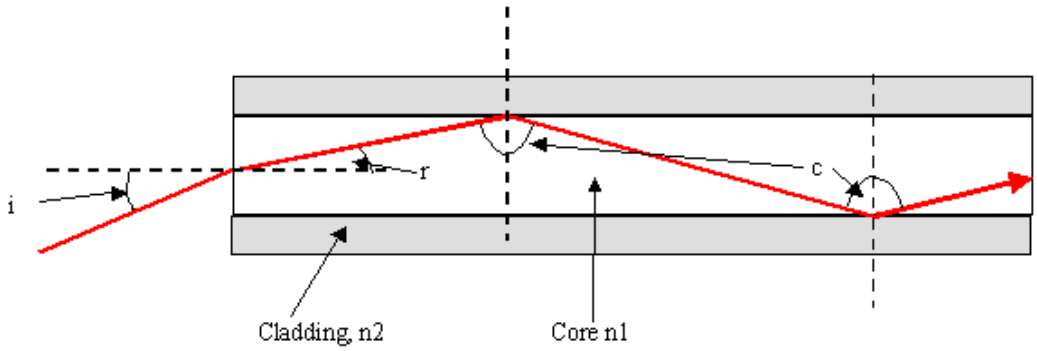


Figure 50 Passage of light through an optical fibre

Questions 14B.06.3 to 14B.06.5 help you to revise total internal reflection.

The maximum angle of incidence in Questions 14B.06.3 to 14B.06.5 was about 38°. Anything above that angle, the refraction will make the light ray strike the side of the glass at less than the critical angle. This maximum angle of incidence is called the **half angle**.

The analysis above works for straight fibres. If the fibre is bent sharply, then light will escape. However as long as the radius of the bend is about 20 times the diameter or more, the light loss will be insignificant.

14B.062 Image formation

Optical fibres are bundled together into a **fibre optic light guide** and the whole bundle is held together by a **sheath**. In the short length that is used in an endoscope, about 1 metre, the light loss is negligible. Each fibre carries the light that is incident on it independently of the other fibres. To transmit a picture, the fibres in a bundle have to be kept in the same relative positions with a smooth square surface so that each fibre is contributing to the formation of the image. This is called a **coherent** bundle (*Figure 51*).

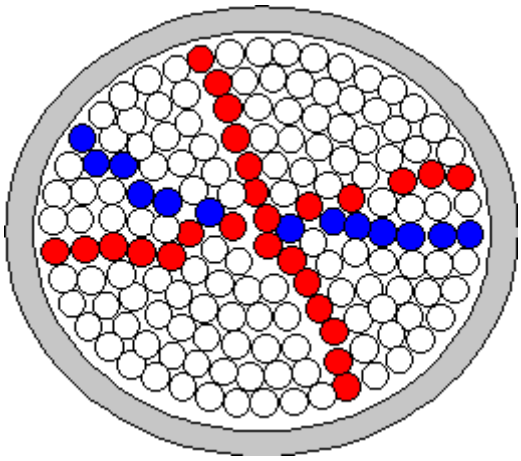


Figure 51 Coherent bundle in a fibre optic light guide

You can see how each fibre contributes to the picture. Each fibre is about 10 mm across. The more fibres there are, the more detailed picture that is possible. A good quality endoscope will have about 40 000 fibres packed into a bundle about 3 mm across.

The endoscope has to have a light source for the doctor to see what's going on. The fibres for this do not have to be coherent (hence non-coherent), and are somewhat wider, about 30 mm. This ensures more efficient transfer of light. Also it is cheaper to produce.

14B.063 The Endoscope

The first endoscope was invented by a German doctor, Adolph Kassmaul (1822 – 1902), at the turn of the twentieth century, but being non-flexible, was not much use. A flexible device was made by another German, Rudolf Schindler (1888 – 1968), but it was still not very flexible unless the patient was contorted to suit the instrument. Schindler's wife did the manipulation of the patient in the early days. The endoscope became a much more practical instrument in the 1960s with the invention of the optical fibre.

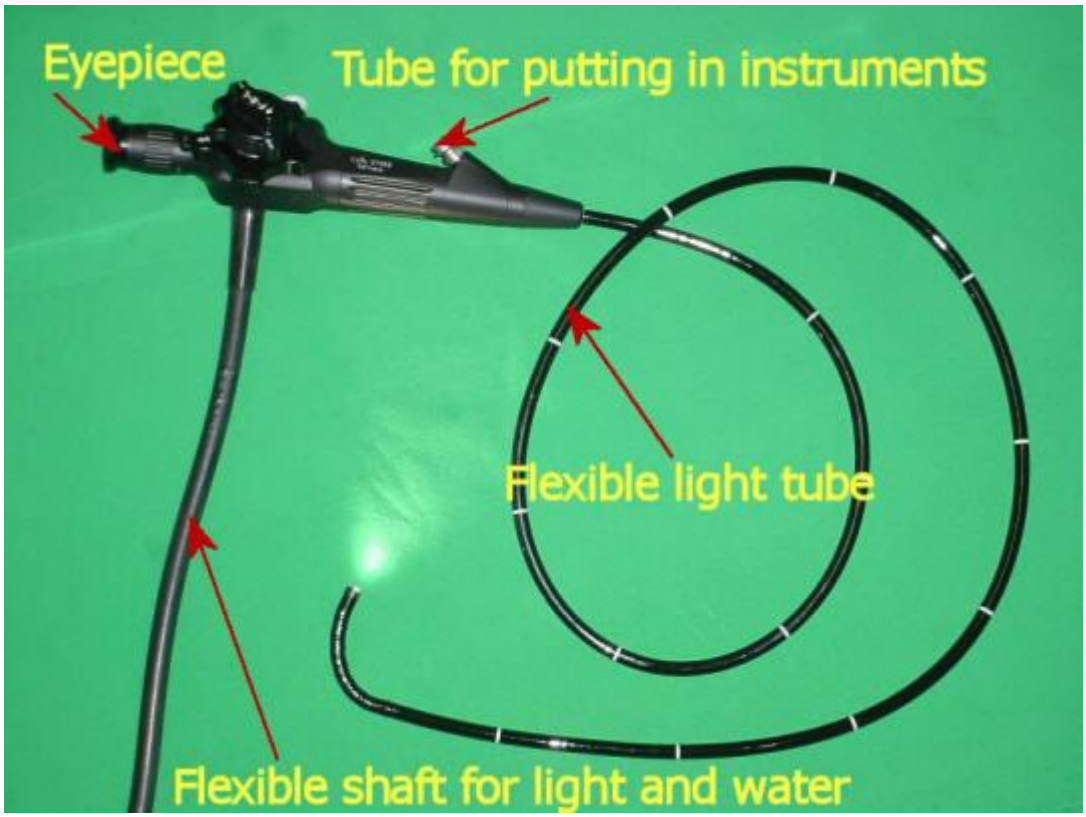


Figure 52 A modern endoscope

The modern endoscope has (*Figure 52*):

- Two light channels to illuminate the area of interest.
- An image channel to enable the doctor to see what's happening.
- An instrument channel through which instruments can be placed
- An air or water channel to wash the area being operated on.
- Control cables to make the business end move.

A range of special instruments can be inserted which enables doctors not only to see inside to make an accurate diagnosis but also operate on the diseased area. This might be to:

- cut out diseased tissue.
- take a biopsy.
- seal a site of bleeding with heat (cauterising).
- remove an object that is causing an obstruction.

At the eye piece end, there can also be attached:

- a TV camera
- a still camera
- a movie camera

The endoscope has also enable doctors to carry out **keyhole surgery** where surgical treatments can be applied without having to make major incisions. This leads to less complications and more rapid recovery of the patient.

The latest development is to have a **video camera** at the end of the endoscope.

14B.064 Lasers

LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. If a photon of the right wavelength hits an excited atom of certain materials, it can stimulate the emission of a second photon of exactly the same wavelength and phase as the first. If enough atoms are excited, the photons can stimulate further emissions of further photons, all travelling in the same direction. At one end of the material there is a mirror that totally reflects the photons, while at the other is a mirror that partially reflects the photons. *Figure 53* shows a LASER.

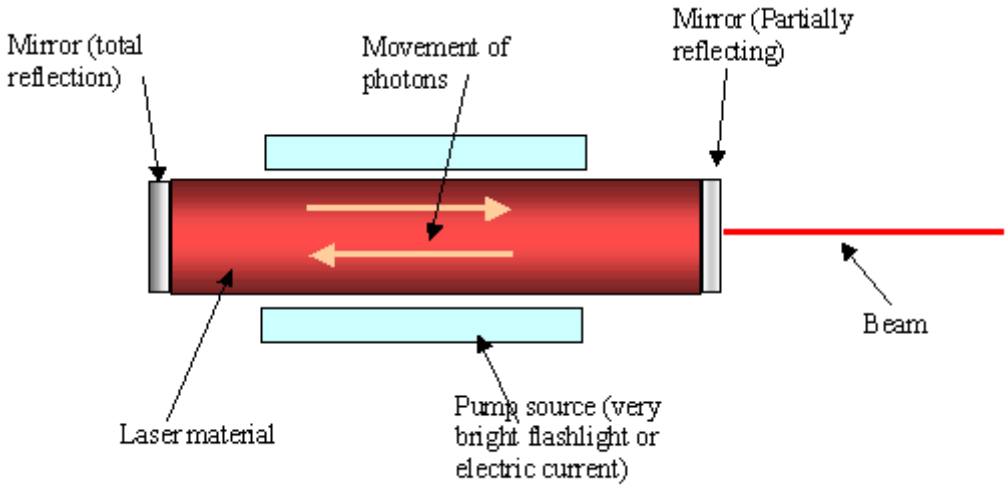


Figure 53 Layout of a LASER

The properties of laser light are:

- monochromatic
- coherent (in phase, of the same amplitude).

Lasers can produce **continuous** light. **Pulsed light** is achieved not by turning the laser on and off, but by placing a shutter between the two mirrors.

The table shows some of the continuous lasers that can be used in medicine:

Laser	*Wavelength (nm)*	*Power (W)*	*Fibre Transmission*
CO_2	10600	0.5 – 50	No
Nd-YAG	1064	0.5 – 100	Yes
Argon	488 or 514	1 – 10	Yes
Dye	Tuneable	0.05 – 5	Yes

This table shows some of the pulsed lasers:

Laser	*Wavelength (nm)*	*Pulse duration*	*Energy per pulse (J)*	*Fibre Transmission*
Nd-YAG (QS)	1064	nanoseconds	0.1 – 1	No
Nd-YAG	1064	microseconds	0.1 – 1	Yes
Dye	tuneable, visible	microseconds	0.01 – 0.1	Yes
Excimer	UV	nanoseconds	0.01 – 0.1	Yes

Skin will absorb laser light:

- far infrared is absorbed to about 0.1 mm.
- near infra-red, a couple of cm.

The black pigment melanin increases the absorption. The pigment haemoglobin in blood absorbs the blue green light of an argon laser. **Tissue damage** arises because the water in cells boils and proteins are denatured, just like when they are cooked.

Lasers are often used with **optic fibres** to guide the laser to the precise location it is needed.

Tissue damage is reduced by using a pulsed laser, but there are difficulties sending high powered laser pulses down glass fibres as they will shatter due to thermal shock.

Lasers are used to cut away diseased material. The **Carbon Dioxide laser** can be used with very delicate tissues such as the brain. However, it cannot be used with optical fibres as the far infra-red radiation is absorbed by the fibres. The **argon laser** can be used to spot weld the retina if it becomes detached. It also used to remove birthmarks. Lasers can be used to seal up (cauterise) bleeding wounds and ulcers.

There is work being carried out in which a chemical is injected into a tumour and can be activated by laser light to become toxic to the tumour, but not to the rest of the body. This is called **phototherapy**.

14B.065 Safety Issues with Lasers

Lasers are potentially dangerous. Hazards arise from:

- the beams causing severe and deep burns
- shining a beam into the eye will cause blindness.

Therefore, strict rules have to be enforced so that persons using lasers are fully trained and competent in their use.

14B.066 Magnetic Resonance Scanner

This machine is used widely in hospitals to carry out **magnetic resonance imaging** (MRI). The technique was invented by Peter Mansfield (1933 – 2017), Professor of Physics at the University of Nottingham, and Paul Lauterbur (1929 – 2007), an American chemist. A picture of the machine is shown below (*Figure 54*):

Figure 54 An MRI scanner (Image from Ptrump16 – Wikimedia Commons)

MRI scanning is a very safe technique, as no ionising radiation is used. However, it can be quite claustrophobic due to the large size of the magnet (on the right hand side of the picture). The space within the magnet is about 60 cm in diameter. You are not expected to know about the precise constructional details of the machine. However, it is worth saying that the machine produces a static magnetic field, the strength of which is up to 3 Tesla. There is also an alternating magnetic field of about 60 MHz. The magnets are superconducting, so have to be kept very cold.

The key to MRI is that the human body is, in effect, a bag of seawater. The MRI scanner uses properties of the **water molecule** for it to work. You will be familiar with the water molecule as *Figure 55*:

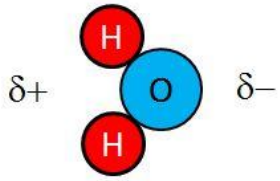


Figure 55 Water Molecule

The water molecule is **polar**. That means that the electrons tend to move towards the oxygen atom, leaving the hydrogen atoms with a slight deficiency of negative charge. Therefore, it has a slight positive charge. The MRI scanner does not interact with the oxygen atom, or the electrons around the hydrogen atom. It does interact with the protons (hydrogen nuclei). To distinguish the protons in the hydrogen and the oxygen, we will call the protons in the hydrogen the **hydrogen nuclei**.

The hydrogen nuclei have the quantum property called **spin**. This allows them to interact with magnetic fields. Normally the hydrogen nuclei are randomly oriented, just like domains in a magnet. So, there is no magnetisation. The MRI scanner uses a combination of strong magnetic fields and radiofrequency electromagnetic radiation to "irritate" them.

An Analogy

I will use an analogy to show the idea. Consider a number of teenage students doing a weekend residential. They are in a large tent. They are asleep. It is dark, so you can't see them (*Figure 55*).

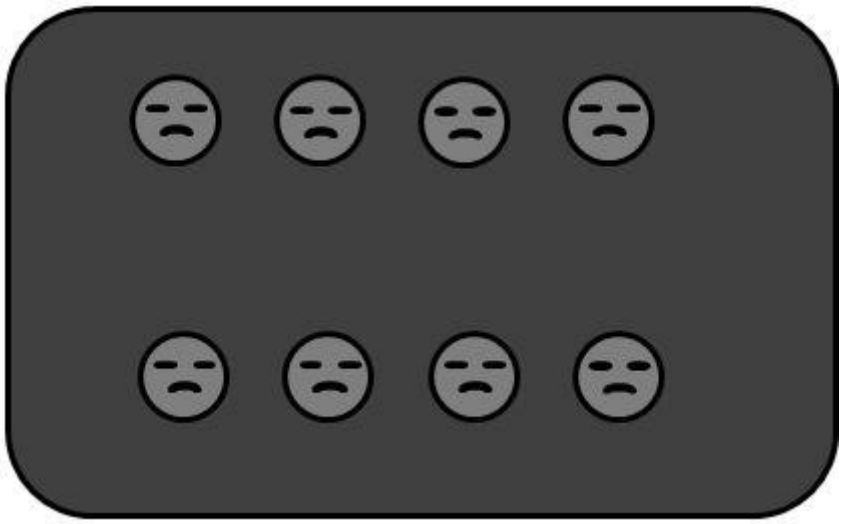


Figure 56 An analogy to explain MRI

It is early in the morning. The students will have to get up early to undertake their day's challenge. Then their teacher comes round and shouts in a rasping voice, "Morning! Wakey, Wakey! Rise and shine!" (*Figure 57*).

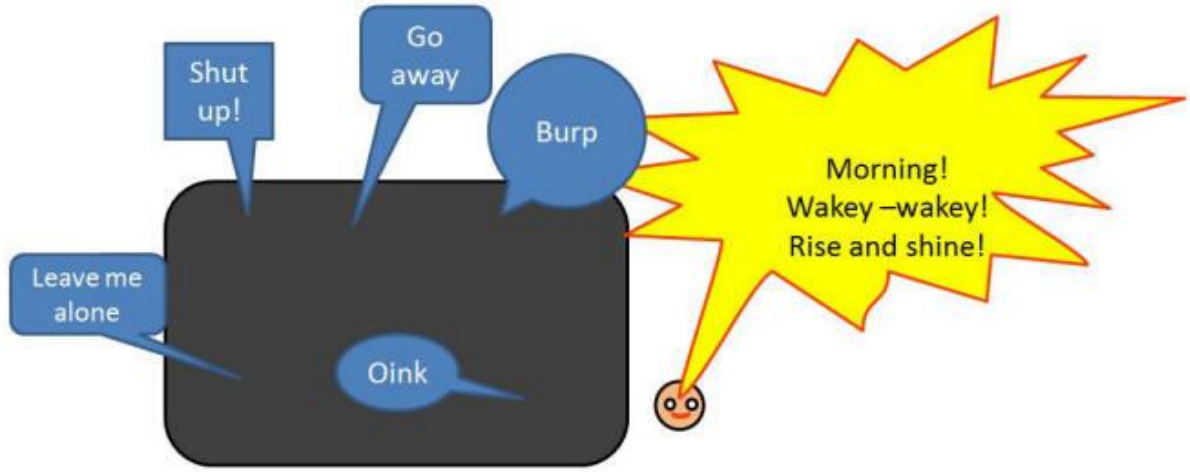


Figure 57 Analogy to explain MRI Part 2

It is dark, so you can't see the students. But you can hear them and locate them. Not all the students reply, but you can see that these students have replied (*Figure 58*).

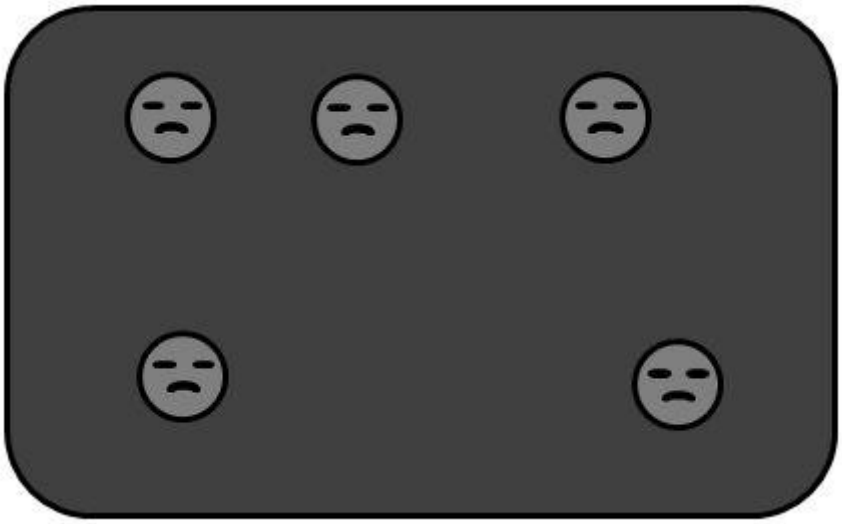


Figure 58 MRI analogy Part 3

Because the hydrogen nuclei have **spin**, they orient themselves along the magnetic field lines, as iron filings do. They act as little compasses. Most will align with the magnetic field. These are nuclei with **low energy**, marked with the letter, 'L'. Unlike the compasses, however, which all align in direction of the magnetic field, a minority of nuclei orient themselves in the **opposite direction** to the magnetic field. This requires more energy, so the nuclei are called **high energy** and are marked in the diagram with a letter 'H' (*Figure 59*). Spin is a quantum phenomenon that works on **probability**. Therefore, there is a probability that any nucleus can achieve the higher energy level.

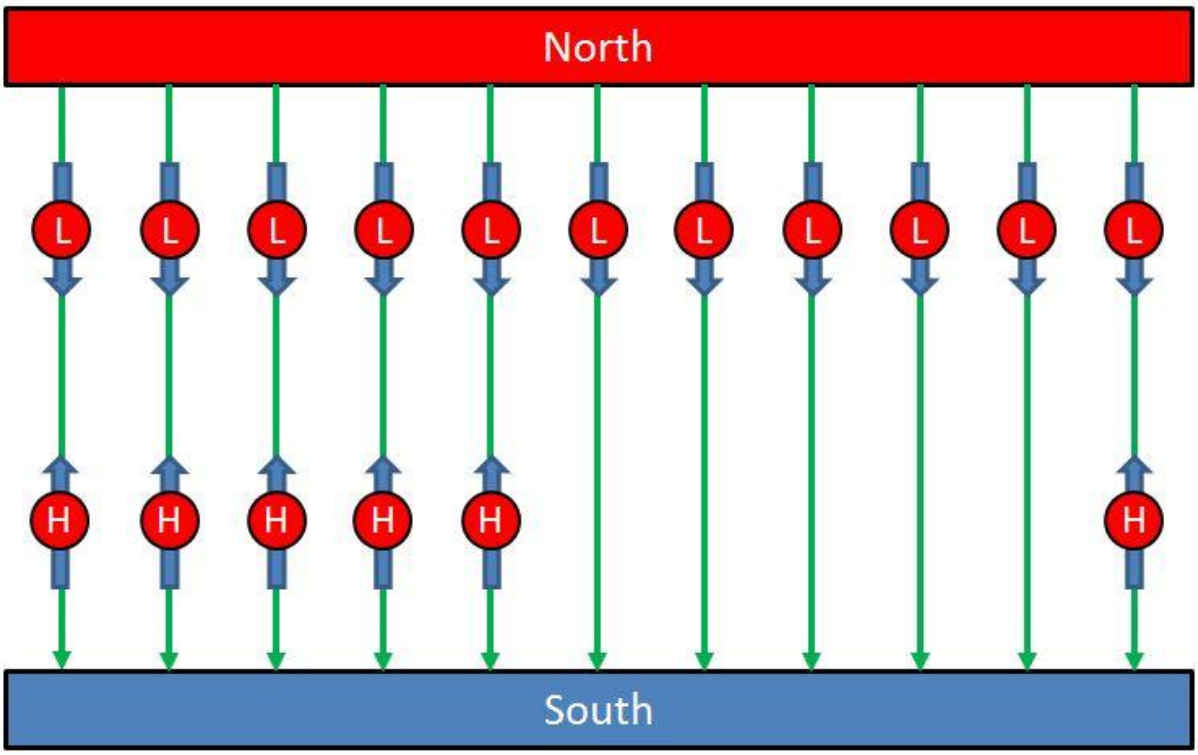


Figure 59 High and low energy nuclei aligning themselves with the applied magnetic field

There are more low energy nuclei than high energy nuclei. So, we see (*Figure 60*):

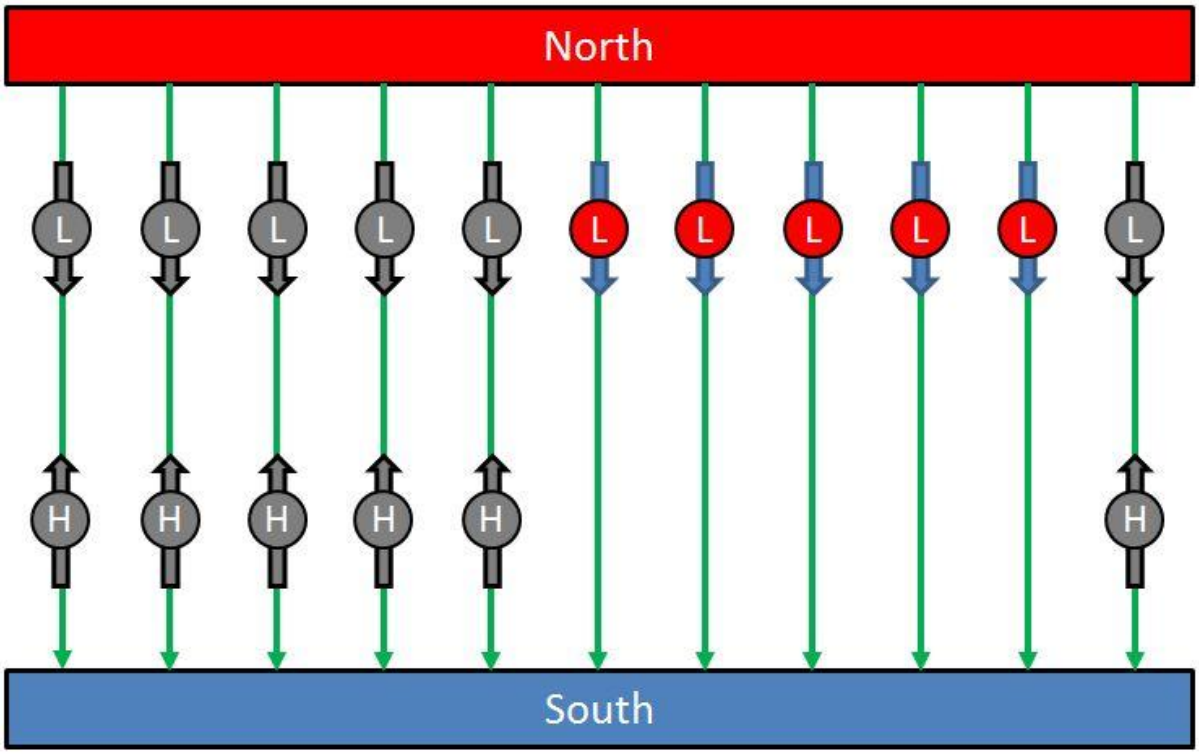


Figure 60 More low energy nuclei than high energy

At the moment, all we have is a physics curiosity. Now we have to do something to the unpaired low energy nuclei. They will respond to an alternating magnetic field, which is provided by a coil that carries a radio frequency current. The field coils that do this are sometimes called the **gradient field coils** (*Figure 61*). The radiofrequency current is in pulses; it NOT continuous.

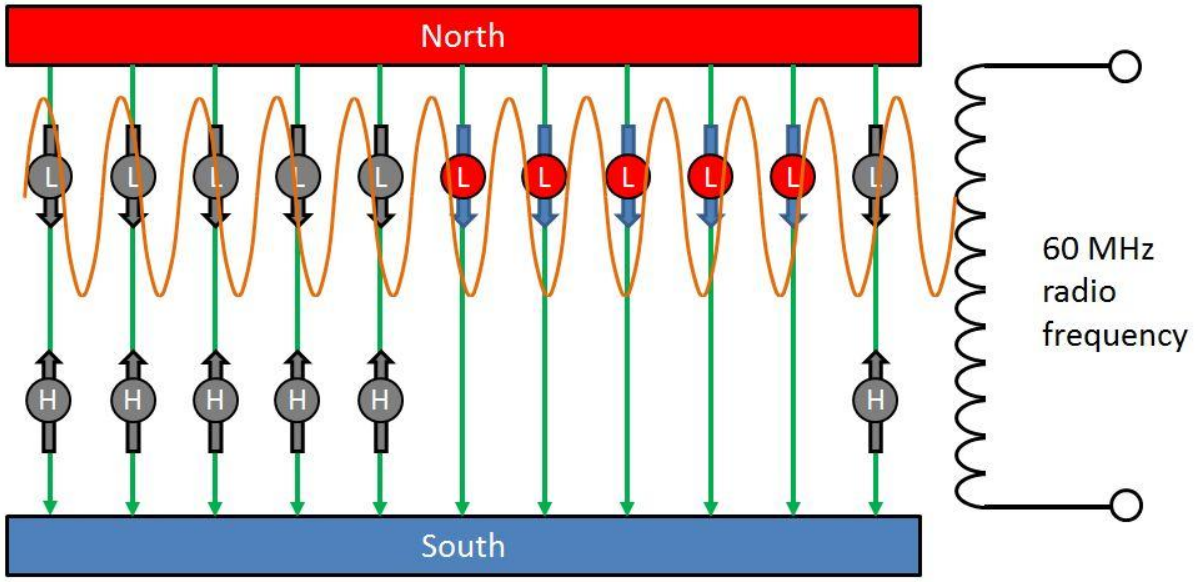


Figure 61 A radio frequency applied to the low energy nuclei

This makes the hydrogen nuclei oscillate like this:

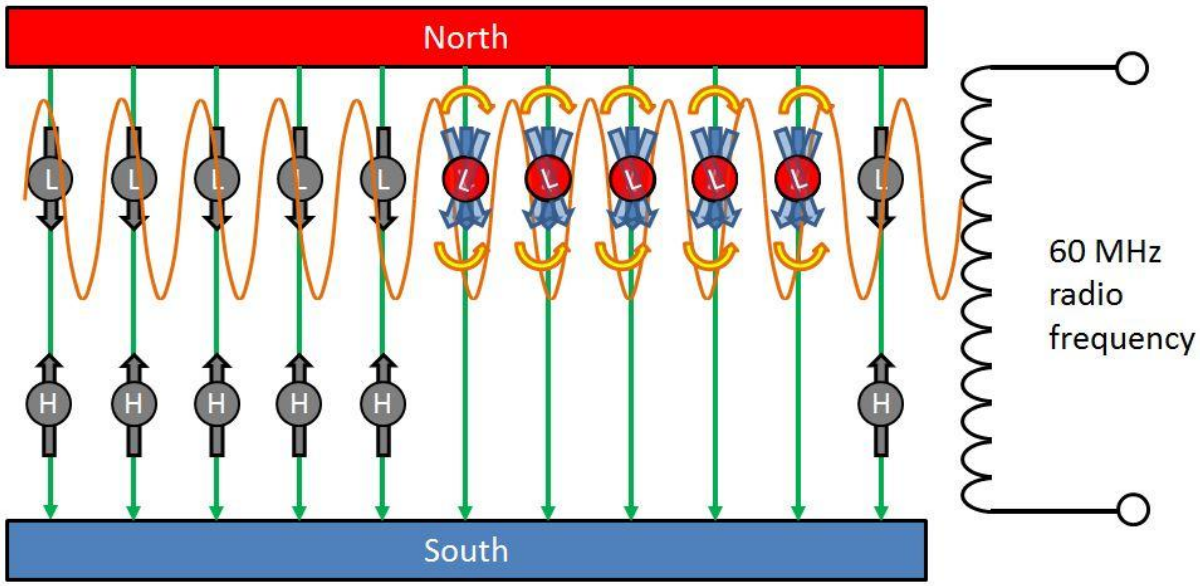


Figure 62 Low energy nuclei oscillating as a result of an applied radio frequency

At a certain frequency, the little magnets gain sufficient energy to flip over like this (*Figure 63*):

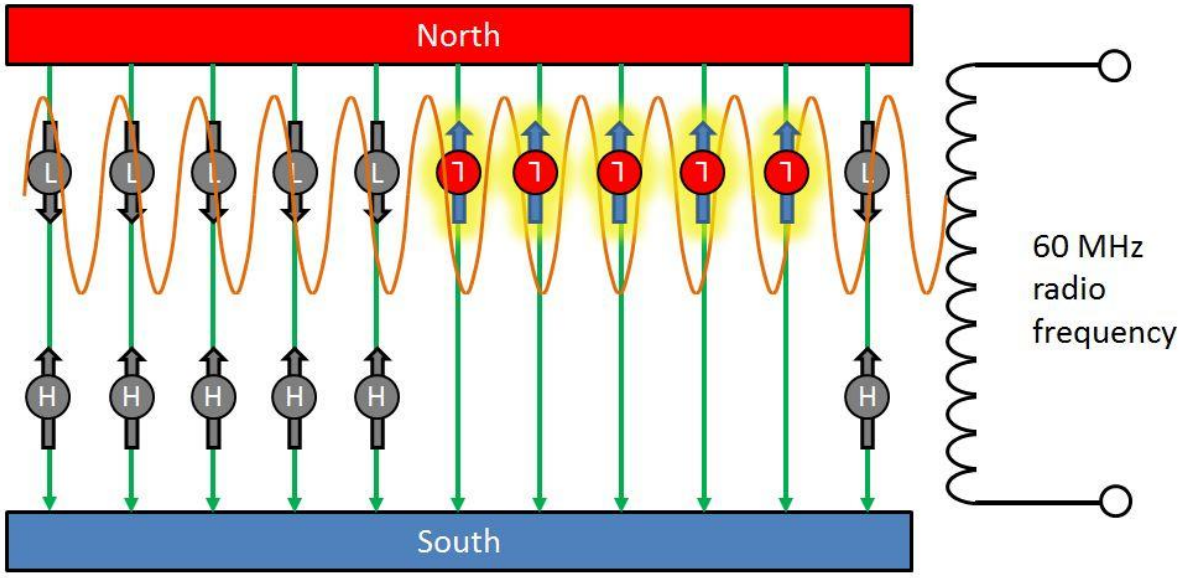


Figure 63 The low energy nuclei flip over

Then the radio frequency is turned off (*Figure 64*):

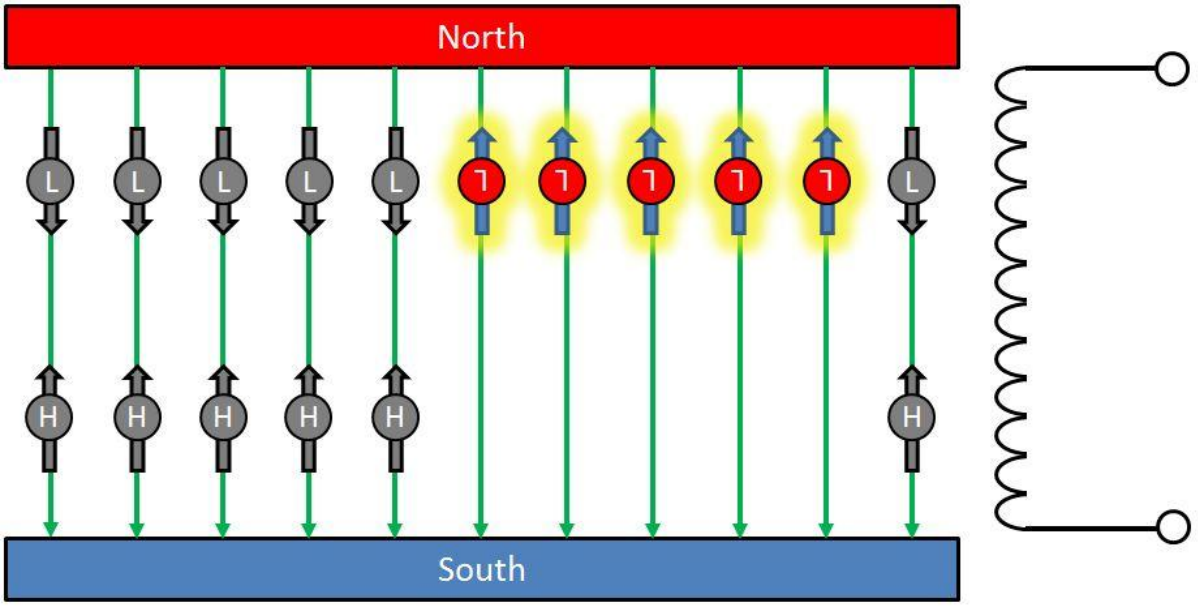


Figure 64 The radio frequency is turned off, leaving the nuclei and an excited state

Immediately the little magnets flip back to where they were, and give off radio frequency energy (*Figure 65*):

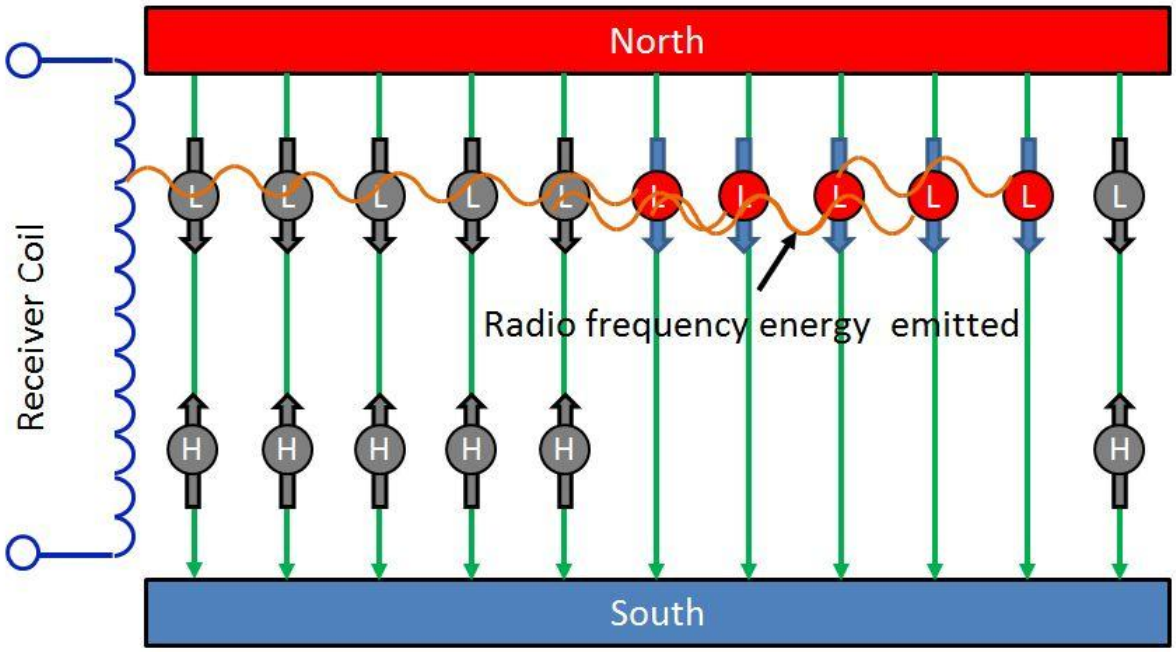


Figure 65 The nuclei immediately lose their excited energy as radio frequency energy and flip back to where they were

The radiofrequency current is in pulses; it NOT continuous. This enables the receivers to pick up the waves emitted by the hydrogen nuclei. The time taken for the hydrogen nuclei to flip back from the high to the low energy state is called the **de-excitation relaxation time**. You will not be asked questions on this.

The receiver coil picks up the radio waves from the hydrogen nuclei (*Figure 66*):

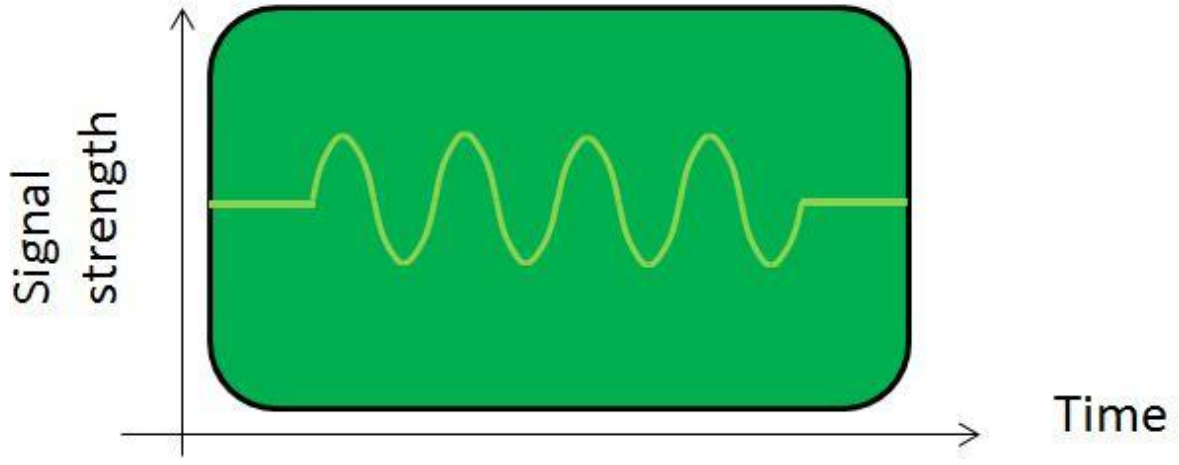


Figure 66 Output RF signal

The signals picked up by the receiver are not displayed like this CRO trace. They are processed using powerful computers to give out an image that can be looked at by the doctors. Here are some images from an MRI scan (*Figure 67*):

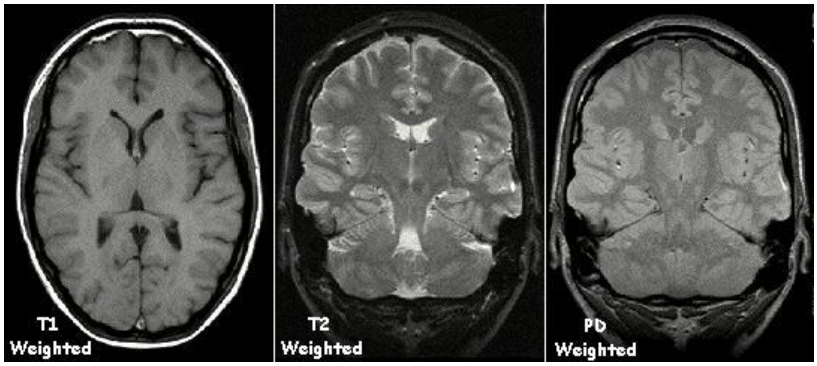


Figure 67 Image from an MRI scan (Source: KieranMaher, Wikimedia Commons)

Often a fluid is injected to enhance the contrast to enable more detail to be seen in the pictures. Normal and abnormal tissues respond slightly differently to the dyes, so different signals are given out.

If all the hydrogen nuclei in a particular area are made to resonate at the same time, the image could end up quite messy. The way around this comes from the fact that the resonant frequency of the nuclei is linked to the magnetic field strength. The higher the magnetic field strength, the higher the resonant frequency. So, we can adjust the magnetic field strength by have a flux density of 1.5 T in the area of interest and reducing the magnetic field strength to 1.0 T in the adjacent areas. The hydrogen nuclei will not resonate in the adjacent areas. This enables pictures to be made of the body in the form of **slices** a few millimetres thick. The picture shows the idea (*Figure 68*).

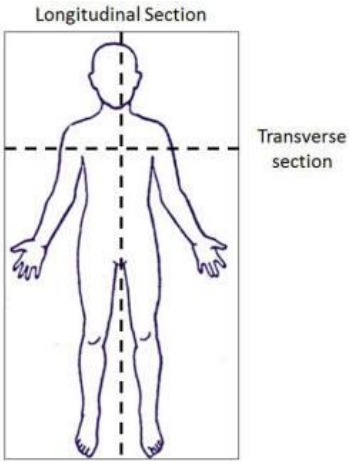


Figure 68 Longitudinal and transverse sections for a patient

Although the slices are normally perpendicular or **transverse** to the body, they can be at any angle the doctors want. It's like slicing bread. Usually, bread is sliced at 90 degrees, but it doesn't have to be.

Here is a picture of a healthy 20-year-old man. The slice has been taken lengthwise (or **longitudinally**). This is shown in *Figure 69*.

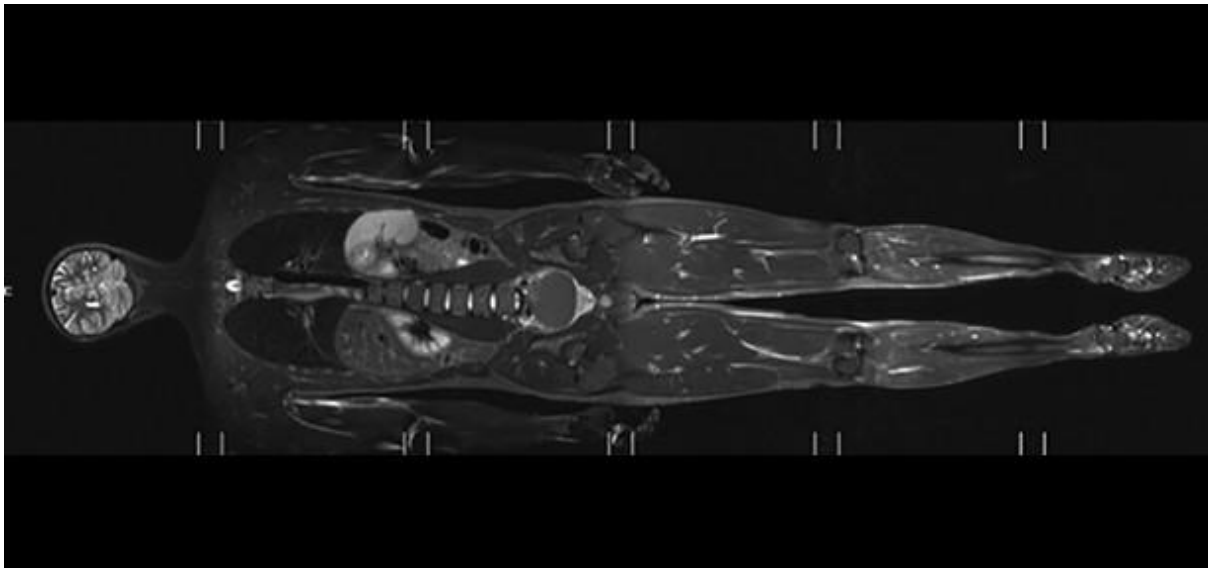


Figure 69 Longitudinal section of a healthy young man (Image from Cincinatti Children's Hospital)

The data from the slices can be combined to make a three-dimensional image of a particular organ.

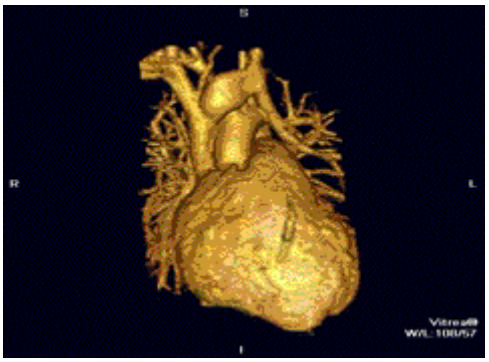


Figure 70 3-dimensional image of a heart (Image by Jccmoon, Wikimedia Commons)

14B.067 Benefits and Drawbacks

Benefits of MRI

- It is non-invasive.
- No ionising radiation is used.
- Very clear images are formed.
- Images can be obtained from any direction.
- Scans can cover large areas of the body.

Disadvantages of MRI

- It is quite expensive.
- It can be a claustrophobic experience, and the machine is quite noisy in operation.
- It does not detect all cancers.
- Metal implants can be affected by the very strong magnetic fields.
- It cannot be used with obese (very fat) patients (as they may not fit into the space in the magnet).

When a patient is about to have an MRI scan, it is very important that metal objects are removed from the room. This includes keys, jewellery, and watches. Injuries have occurred when metal objects have been attracted to the very powerful magnet. Other injuries have been caused by metal implants, for example screws used to treat bad leg fractures. Also, shrapnel embedded from injuries sustained on active service can do a lot of damage.

Burns can happen from rings on fingers, since the ring can act as a perfectly good secondary coil to the radiofrequency coil. Therefore, eddy currents are induced, which would lead to a marked heating effect in the ring.

Careful management of the procedure will minimise such risks.

14B.068 The Larmor Frequency

(*Welsh and Eduqas*)

The **Larmor Frequency** (strictly speaking the Larmor Constant), named after Joseph Larmor (1857 – 1942), an Irish physicist, is used as a **constant of proportionality** in NMR.

It has the value of 42.6×10^6 Hz T^{-1}. The units may be quoted as MHz T^{-1} (so watch out for this).

It is at this ratio that the hydrogen nuclei **resonate** in a magnetic field, which is the crucial part of NMR. If the value of the magnetic field is 1.0 T, the frequency is 42.6 MHz. We can use any value of magnetic field.

The resonant frequency for protons is worked out the equation:

$$f = 42.6 \times 10^6 B \quad \text{..... Equation 18}$$

We can show the proportionality on the graph (*Figure 71*):

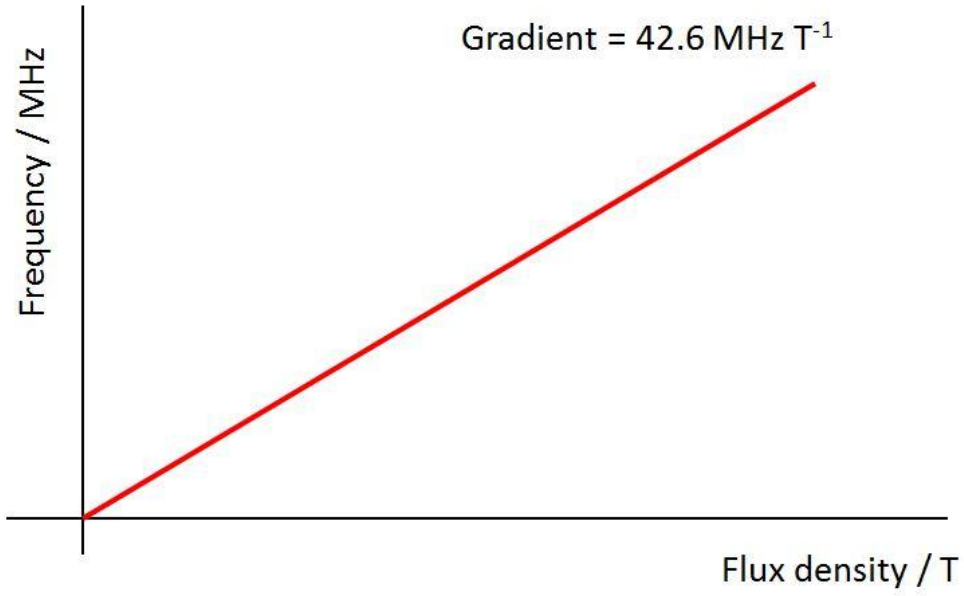


Figure 71 Graph showing proportionality between frequency and flux density

Other nuclei have a different ratio.

Derivation (Extension only)

You are NOT expected to derive the equation you have used to work out the resonant frequency. I have included this argument to show how the constant of proportionality arises. You need to be aware of the idea of **angular momentum**. Like an object moving a straight line, spinning objects possess angular moment. You can find out more about this in Topic 14C Engineering Physics.

In the discussion on NMR above, we considered the hydrogen nuclei in two dimensions only, i.e. they waggled forwards and backwards. In three dimensions, they spin round rather like a gyroscope or spinning top.

Consider an electron of charge e in a magnetic field of flux density B. The electron rotates on an **axis of spin** (just like the Earth rotates on an axis of spin). If the electron is in a magnetic field of strength B, the axis of spin will trace a **circular path** as shown in the diagram (*Figure 72*)

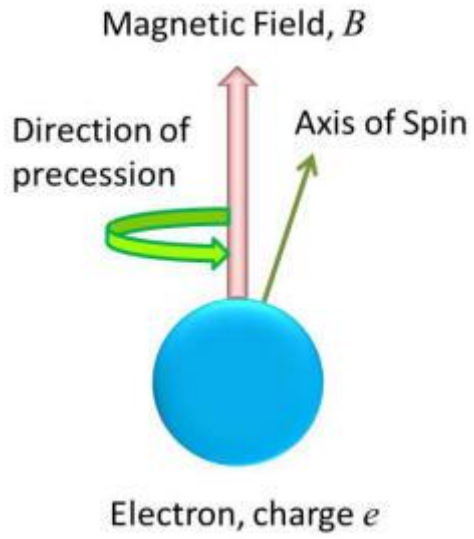


Figure 72 Spin of an electron in a magnetic field

The change in the orientation of the axis of spin is called the **precession**. The proton can be considered to be a tiny **magnetic dipole**. The orientation of the axis of spin is also the orientation of the magnetic dipole. Therefore, the axis is considered to undergo a **magnetic moment**. As the moment is acting on an object with rotational moment, we can say that it has angular momentum, L, and is subject to a torque, τ, from the external

magnetic field, B. (The symbol τ is "tau", a Greek lower case letter 't', and is used as the Physics code for torque.)

These two quantities are related by a simple equation:

$$\tau = \mu B \quad \text{..... Equation 19}$$

The term μ is the **magnetic dipole moment**.

The magnetic dipole moment is related to the angular momentum, L, and the magnetic field by:

$$\tau = \gamma L B \quad \text{..... Equation 20}$$

Therefore:

$$\mu B = \gamma L B \quad \text{..... Equation 21}$$

The term γ (gamma) is called the **gyromagnetic ratio**. The magnetic field term cancels out. Therefore:

$$\mu = \gamma L \quad \text{..... Equation 22}$$

The gyromagnetic ratio is the ratio of the magnetic moment to the angular momentum.

It can be shown that for any precessing (wobbling of the axis of rotation) particle:

$$\gamma = \frac{eg}{2m}$$

..... Equation 23

Where:

- γ - gyromagnetic ratio (C kg^{-1}).
- e - charge of the precessing particle (C).
- m - mass of the precessing particle (kg).
- g - the **g-factor**, a dimensionless factor that depends on the particle.

Particle	g-factor
Electron	2.002
Neutron	3.826
Proton	5.586

Worked Example
What is the gyromagnetic ratio of a proton?

Mass of a proton = 1.673×10^{-27} kg.
g-factor = 5.586

Answer
Use:

$$\gamma = \frac{eg}{2m}$$

$\gamma = (1.602 \times 10^{-19}\ C \times 5.586) \div (2 \times 1.673 \times 10^{-27}\ kg) =$ **$2.674 \times 10^{8}\ C\ kg^{-1}$**

The angular momentum vector precesses at an angular velocity of ω rad s^{-1}. This is often called the **Larmor frequency**. It is an angular velocity. It can be shown that the Larmor frequency is given by the relationship:

$$\omega = \gamma B \quad \text{..... Equation 24}$$

So, we can write:

$$\omega = \frac{egB}{2m}$$ ..... *Equation 25*

From circular motion, we know that:

$$f = \frac{\omega}{2\pi}$$ ..... *Equation 26*

Therefore:

$$f = \frac{egB}{4\pi m}$$ ..... *Equation 27*

Worked Example

What is the Larmor frequency of a proton at a magnetic field strength of 1.0 T?

Mass of a proton = 1.673×10^{-27} kg.
e = 1.602×10^{-19} C
g-factor = 5.586

Answer

Use:

$$f = \frac{egB}{4\pi m}$$

f = $(1.602 \times 10^{-19}\ C \times 5.586 \times 1.0\ T) \div (4 \times \pi \times 1.673 \times 10^{-27}\ kg)$ = **42.57×10^6 Hz**

Therefore, the Lamour frequency for protons gives the equation:

$$f = 42.6 \times 10^6\ B$$ ..... *Equation 28*

Questions

Tutorial 14B.06

14B.06.1

What is the condition needed for total internal reflection?

14B.06.2

What would happen if the light ray in the core hit a boundary between the core and a material of higher refractive index? Would it matter?

14B.06.3

An optical fibre has a core refractive index of 1.55, and the cladding refractive index is 1.42. What is the critical angle?

14B.06.4

What is the angle of refraction that gives this critical angle? (Question 14B.06.3)

14B.06.5

What is the angle of incidence that gives this angle of refraction, as the light crosses from air into the glass? Refractive index of air is 1.00. (Question 14B.04.4)

14B.06.6

Write down and explain three ways in which a doctor might use an endoscope.

14B.06.7

If there were no mirror at all, more photons could get out. Why should there be a partially reflecting mirror instead of no mirror at all?

14B.06.8

Which laser would you use for the following procedures? Give a reason for each one:

(a) removing a tumour from the brain.

(b) spot welding a detached retina.

14B.06.9

Why doesn't the magnetic field interact with the protons in the oxygen atom?

14B.06.10

Look *Figure 59*. What is the effect of one low energy nucleus combined with one high energy nucleus?

14B.06.11

Look at *Figure* 61. What happens to oscillators at a certain frequency?

14B.06.12

In the discussion on NMR on Page 88, we saw that the radio frequency was quoted as 60 MHz.

What is the magnetic field strength needed to get the hydrogen nuclei to resonate?

Tutorial 14B.07 X-Ray Imaging	
AQA Syllabus	
Contents	
14B.071 Production of X-Rays	14B.072 X-ray Characteristics
14B.073 X-ray Generator	14B.074 Controlling the X-ray Beam
14B.075 Absorption of X-Rays by Tissues	14B.076 Attenuation of X-rays
14B.077 Making X-Ray Images	14B.078 Uses for X rays
14B.079 CT Scan	14B.0710 3-D Images (Extension only)

The use of X-rays for making shadow pictures of bones has been around for over a century. The first X-ray machine was built by a German, Wilhelm Röntgen in the late nineteenth century. Within a few months, the machines were becoming widespread in hospitals. They were also used for amusement as there was little knowledge about the risks involved. Nowadays X-rays are used for:

- looking at fractures in bones.
- looking at teeth to diagnose any decay.
- looking for the shadows caused by tumours and other disease in soft tissue.
- treatment of tumours by radiotherapy.

14B.071 Production of X-Rays

X-rays are photons of electromagnetic radiation produced when a target of heavy metal is struck by electrons travelling at high speed. Only about 1 % of the electrons produce an X-ray photon; the rest is lost in heating up the target. X-rays are produced by:

- slowing the electron down, called **bremsstrahlung** (German for "braking radiation")
- by removing an inner electron. As electrons replace the inner electron, photons are emitted as the electrons undergo transitions from energy level to energy level.

It can be considered that X-ray production is the **photo-electric effect in reverse**. Electrons strike the heavy metal, and the energy is released as a photon. Unlike photo-electron emission, where one photon releases one photoelectron, one electron does NOT release one photon. The process is inefficient. 99 % of the energy in the stream of electrons is transferred to **heat**, and 1 % is transferred to X-ray photons.

Bremsstrahlung is summarised in the diagram (*Figure 73*):

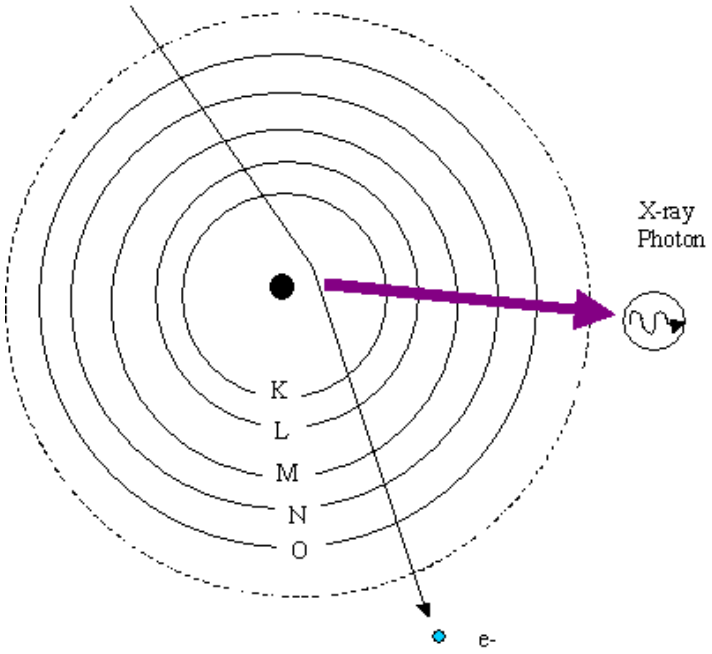


Figure 73 Bremsstrahlung

In **bremsstrahlung** the spectrum of photons is continuous.

When the atom is **ionised** by the removal of an **inner electron**, other electrons fill the space by dropping down the quantum ladder. The photons have a particular energy. This mechanism is shown below (*Figure 74*):

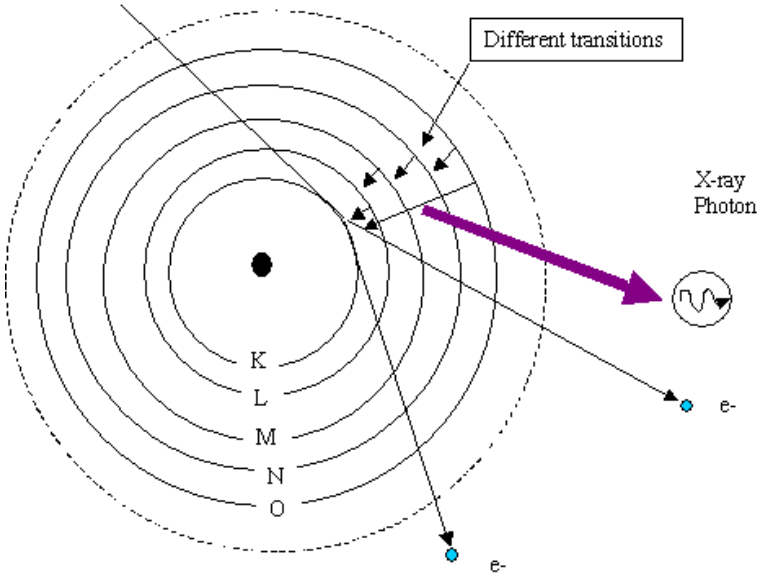


Figure 74 X-ray production

The maximum energy, E_{max}, that can be obtained is when all the energy from the electron is converted into the energy of the photon. So, we can say:

- Kinetic energy = photon energy
- Kinetic energy = charge of electron × voltage

Therefore:

$$eV = hf$$ ..... *Equation 29*

Then we can link this with the wave equation (*Equation 30*)

$$c = f\lambda$$ ..... *Equation 30*

to give us:

$$eV = \frac{hc}{\lambda}$$ ..... *Equation 31*

14B.072 X-ray Characteristics

Ionisation results in a **characteristic** or **line** spectrum. Each line is named by the shell in which the electrons end up. So, if the electron were removed from the K shell, the innermost shell, electrons would fall down the energy ladder giving out a photon of energy consistent with a fall into the K shell. These are called **K-lines**. For **tungsten** as a target material, the accelerating voltage needs to be about 70 kV.

A characteristic is shown below (*Figure 75*):

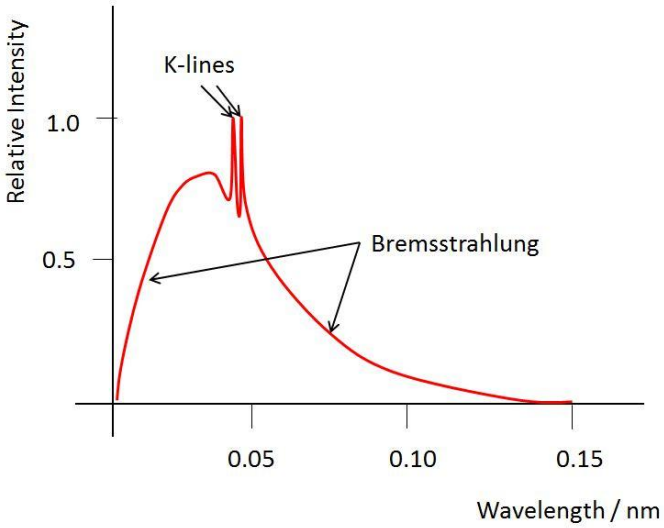


Figure 75 Characteristic showing the intensity of K-lines

Two peaks occur at about 0.45 nm and 0.50 nm.

If we plot the voltage on the horizontal axis we get (*Figure 76*):

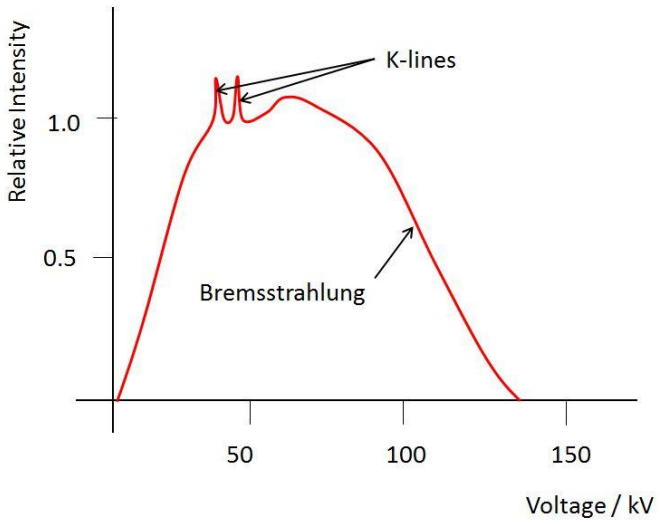


Figure 76 K-lines are produced at about 50 KV

If we reduce the current, we see a graph like this (*Figure 77*):

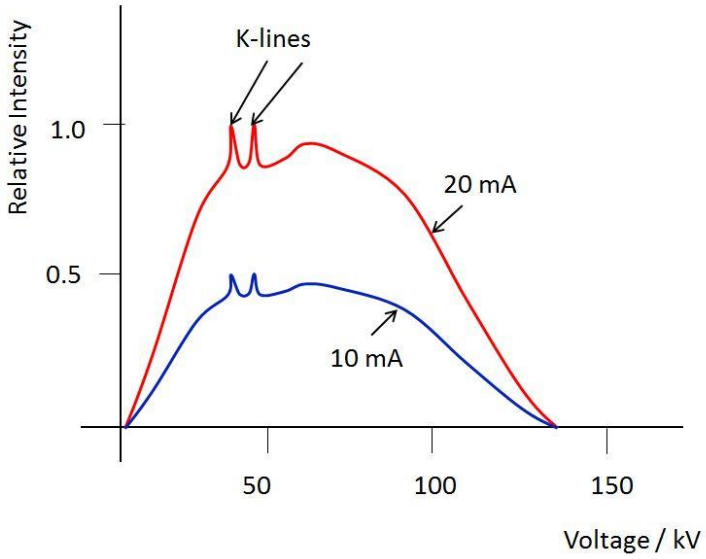


Figure 77 The effect of reducing the current

We should note the following:

- If we halve the current, we get half the intensity. This is reasonable. Halve the number of electrons and we will get half the number of photons.
- The **K-lines** are not affected, nor is the **maximum energy** of the photons. This is because of the **quantum nature** of photon emission.

If we reduce the voltage we would see (*Figure 78*):

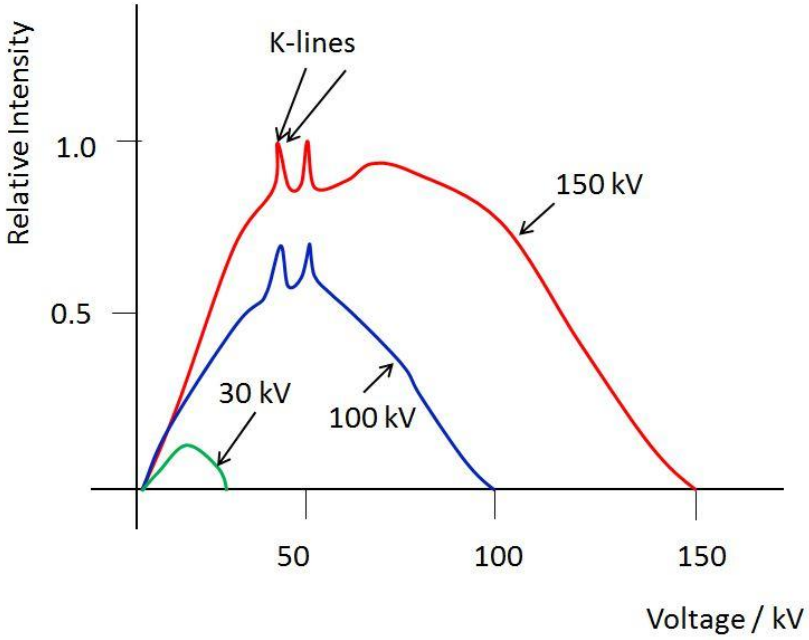


Figure 78 Effect of reducing the voltage

Notice that the K-lines are visible until the maximum energy is less than the energy of the photon energy of the K-lines. At 30 kV, no K-lines are observed. If we increase the voltage, we may well see more characteristic lines.

If we change the **target material**, keeping the voltage the same, we will see a completely different characteristic. As the **proton number is increased**:

- The **maximum energy**, E_{max}**,** remains constant, as it's a function of the voltage.
- the **total intensity** (the area under the graph) will change because there is a **greater probability** of a collision between the incoming electron and the electron shells. The more protons, the more electrons.
- The **characteristic lines** are shifted to higher photon energies.

Low energy X-ray photons are called "**soft X-rays**", while high energy photons are called "**hard X-rays**". About 1 % of the electrons striking the target produce an X-ray photon, so the process is not at all efficient. The rest of the energy is lost as heat.

A filter can be made of a sheet of material that will selectively absorb lower energy photons, so that the beam consists of harder X-rays. The beam is more penetrating.

14B.073 X-ray Generator

The most common X-ray generator is the **rotating anode tube**. It is an evacuated glass envelope, immersed in oil to cool it and surrounded by lead.

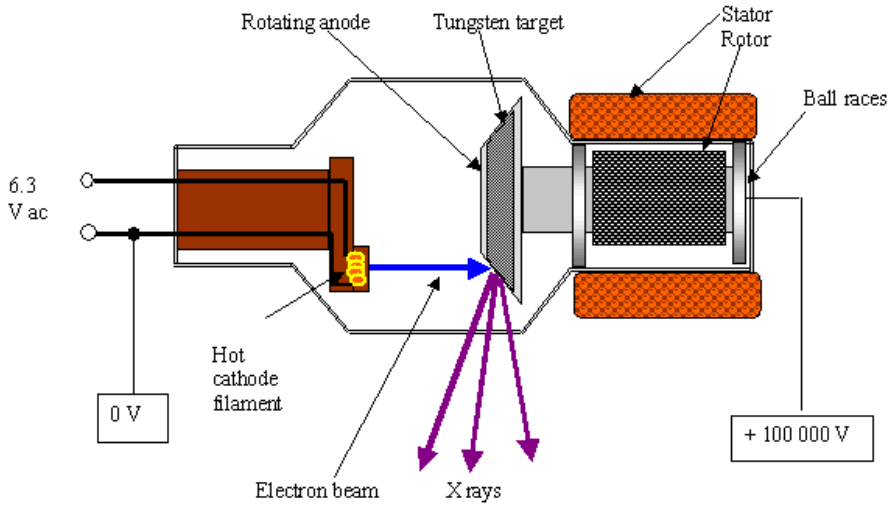


Figure 79 A rotating anode X-ray tube

- Electrons are boiled off the hot filament which glows just like a light bulb. This is called a **hot cathode**, and the process is **thermionic emission**.
- They are accelerated by the anode voltage.
- They hit the target, giving off energy mostly as heat, but 1 % is given off as X-rays.
- The target would rapidly melt, so it is turned by an AC induction motor. The rotor is in the evacuated glass bulb, while the stator (the coils of wire) is on the outside. The cathode spins at 3000 rpm.

The answer to the 14B.07.4 should show how vital the **cooling system** of an X-ray machine is. Actually, if the cooling system were to fail, the machine would be turned off automatically. The X-ray bulbs are precision pieces of kit and extremely expensive. Even a jolt may well break the filament and lead to a massive bill.

14B.074 Controlling the X-ray Beam

Unlike light or electron beams, X-rays **cannot be focused**. So, they can only make shadow images. If you use a small point source of light, you get sharp shadows. If it's a wide source of light, the shadows become fuzzy. Obviously, the doctor wants a sharp shadow.

There are various ways in which an X-ray source can be made into a point source:

- The beam is made narrow by the geometry of the anode to about 17^{o}.
- The beam can be limited by using **apertures**. This can be a simple diaphragm or a cone made from lead.
- Scattering in the tissues can make the picture fuzzy. A grid made of strips of lead will absorb any scattered X-rays.

The diagram shows how the X-ray beam can be directed (*Figure 80*):

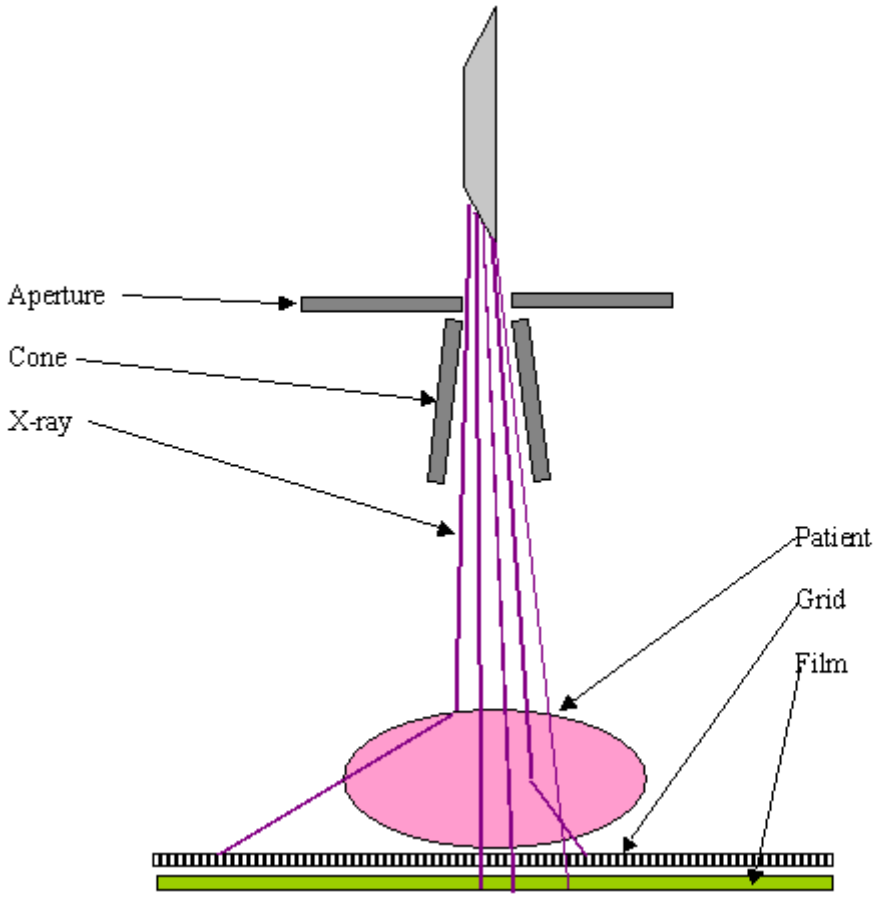


Figure 80 Controlling the X-ray aperture to get precise shadows

The resolution of X-ray images is quite good, about 0.05 mm.

14B.075 Absorption of X-Rays by Tissues

When X-rays pass through materials the energy of the beam is reduced or **attenuated**:

- By scattering. The X-ray photons are reradiated as lower energy photons (*Figure 81*).

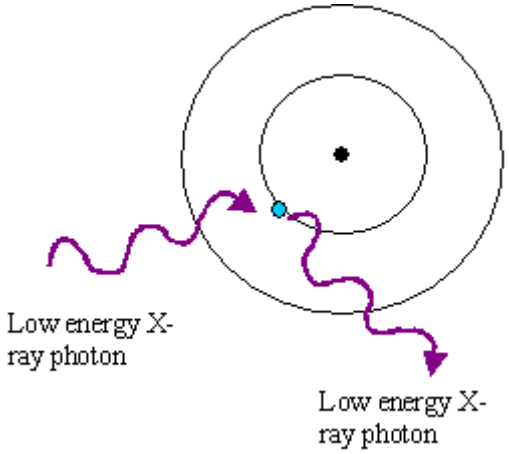


Figure 81 Reradiation as lower energy X-ray photons

- Photoelectric effect where an electron gets ejected. Photons of visible light are given off as the atom comes out of the excited state (*Figure 82*).

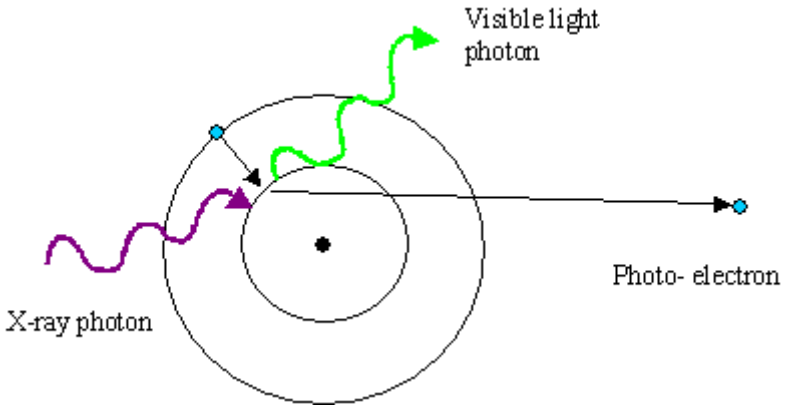


Figure 82 Emission of a photon of visible light and a photoelectron

- **Compton scattering** where both an electron and a lower energy X-ray photon are emitted (*Figure 83*).

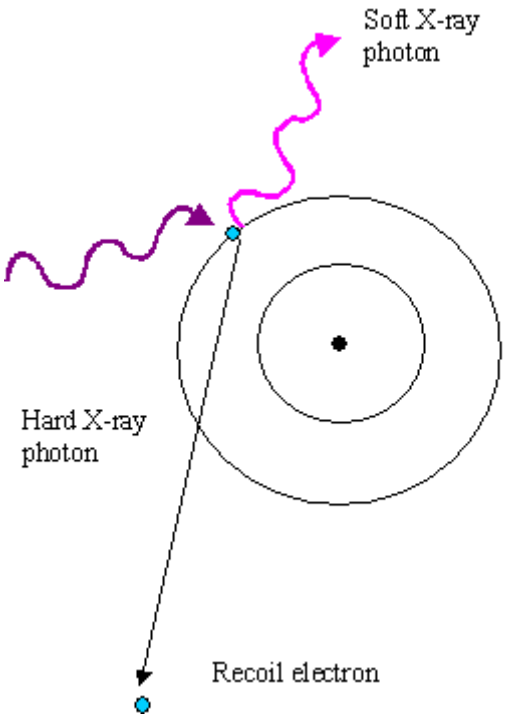


Figure 83 Compton scattering

- Pair production where a very high energy photon interacts with the nucleus of an atom. An electron and a positron emerge, losing their energy by ionisation until the positron is annihilated by an electron, generating two identical photons (*Figure 84*).

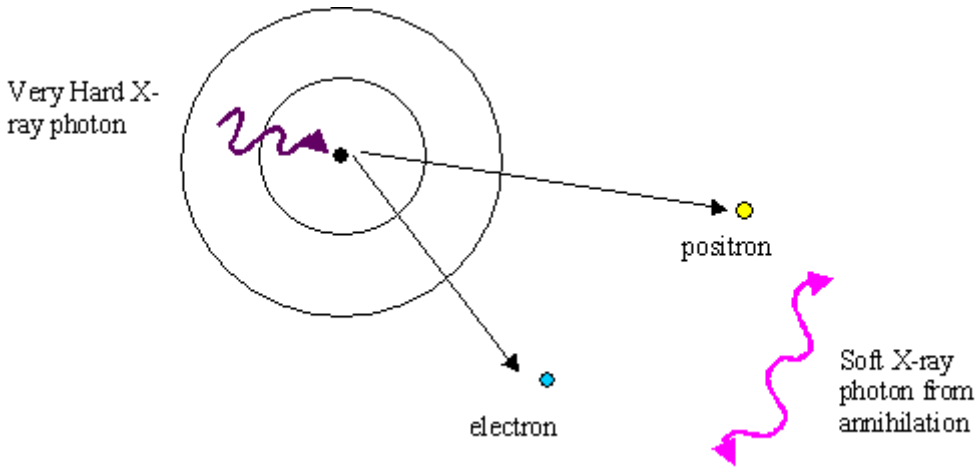


Figure 84 Pair production

Any of these can do immense damage to biological material:

- **Water** is ionised to form **free radicals** that are highly reactive. Free radicals can combine to make **hydrogen peroxide** H_2O_2 which is a powerful oxidising agent and can damage the DNA of the chromosomes.
- At the **molecular level**, enzymes, RNA and DNA are damaged, and metabolic pathways are interfered with.
- At the **sub-cellular level** cell membranes are damaged, along with the nucleus, chromosomes, and mitochondria.
- **Cellular level**, cell division is damaged. Cells can die or be transformed to malignant growth.
- **Tissue and organ damage**. There can be disruption to the central nervous system, death of bone marrow and the lining to the gastro-intestinal system, leading to sickness and death. Cancers may arise.
- Whole animal can die; or life is shortened.
- Populations: mutations can alter the genetic characteristics of populations.

X-ray doses are very carefully controlled and maximum limits are set to minimise the risks to patients. These limits are well below the doses that would cause the least harm. However, the procedures, although very safe, always run a very slight risk of long term harm. So does watching the TV all day.

Bones absorb X-rays which means that good shadow pictures are easy to get. Soft tissue pictures are harder to obtain. They tend to be fuzzy, but there are differences in the absorption by soft tissues. A lung cancer can show up as a shadow on a chest X-ray. The detection of diseased lung tissue is done by X-ray because it's impossible to do with ultrasound.

14B.076 Attenuation of X-rays

In a vacuum (and nearly so in air) the **attenuation** of X rays follows the inverse square law. Double the distance and the intensity reduces by a factor of 4.

In a material there are various different absorption processes going on and the intensity goes down by a constant fraction per unit distance. The calculus process of integration gives us a quantitative relationship:

$$I = I_0 e^{-\mu x}$$

.....*Equation 32*

[I_0 - Intensity at source (W m^{-2}); I - intensity at a certain point (W m^{-2}); μ (mu - a Greek letter 'm')- **total linear attenuation coefficient** (m^{-1}); x - distance (m); e - exponential number = 2.718...]

This means that the intensity goes down **exponentially**, as shown in the graph (*Figure 85*):

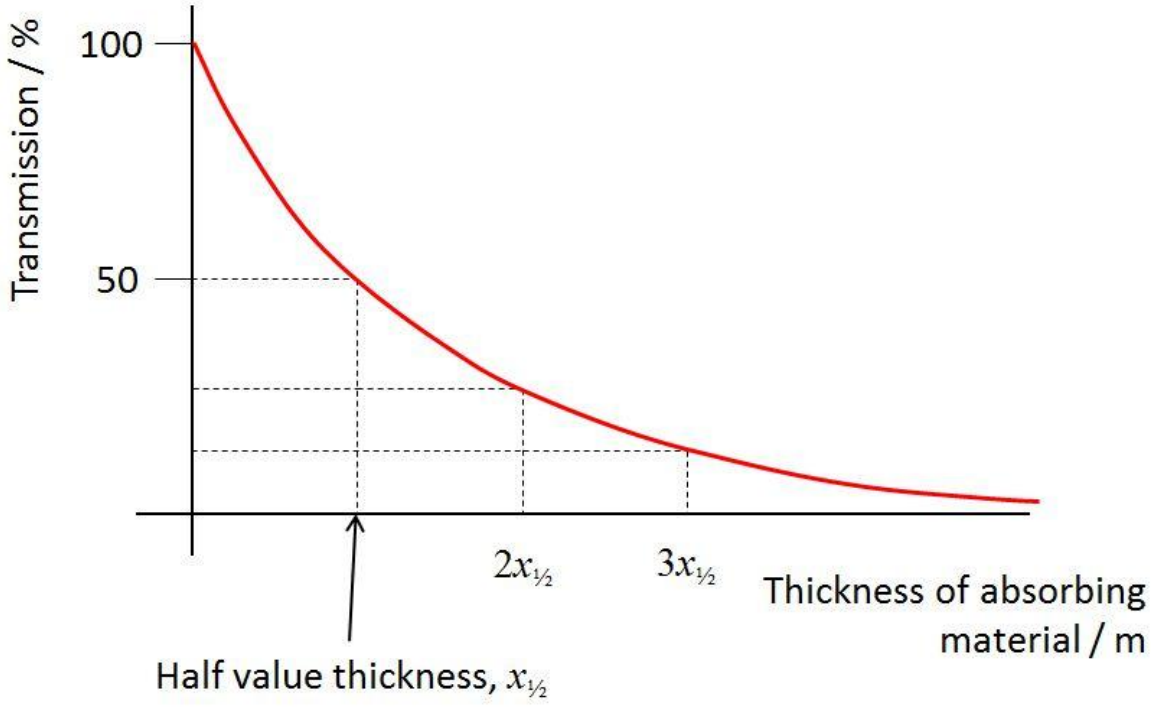


Figure 85 Exponential fall in transmission of X-rays with thickness

Notice the **half value thickness** of the material. It is the **thickness of the material that makes the X-ray intensity 50 % of its original value**. Double it, and the intensity goes down to 25 %. Treble it and it goes down to 1/8 or 12.5 % or the original value.

At the half value thickness:

$$0.5 = e^{-\mu x_{\frac{1}{2}}}$$

..... *Equation 33*

We take natural logarithms. ln 0.5 = -0.693:

$$-0.693 = -\mu x_{\frac{1}{2}}$$

..... *Equation 34*

Rearranging:

$$x_{\frac{1}{2}} = \frac{0.693}{\mu}$$

.....*Equation 35*

For copper the half value thickness is 1 mm, which means that $I = 0.5\ I_0$ behind 1 mm copper.

Worked example What is the linear attenuation coefficient for copper, if its half value is 1 mm?
Answer Use $x_{1/2} = 0.693/\mu$ 1.0×10^{-3} m = 0.693/μ μ = **693 m^{-1}**

Radiographers also use another term, the **mass attenuation coefficient**, μ_m, which is the **attenuation per unit mass of material**. It is linear attenuation coefficient per unit density. The equation is:

$$\mu_m = \frac{\mu}{\rho}$$

..... *Equation 36*

Units are $m^2\ kg^{-1}$.

Bear Trap

- It is very easy to confuse the **total linear attenuation coefficient** with the **mass attenuation coefficient**. Be careful. If you are given the mass attenuation coefficient and you have to use the exponential attenuation equation, you must convert by multiplying by the density.
- That said, I have only ever seen the **half value thickness** of a material given in all the past papers I have looked at. I have never seen a figure for the mass attenuation coefficient.

14B.077 Making X-Ray Images

X-rays themselves are very difficult to focus. Therefore, indirect means have to be used.

The commonest way of getting an **image** from the X-ray machine is a **simple photographic film**. The films used vary in size according to the investigation. For a dental X ray, the film would be about 3 x 4 cm; for a chest X-ray it would be 40 x 50 cm. Unlike a film in a camera, these films are double sided, i.e. they have the emulsion on both sides. The films are developed in the usual way in a photographic dark room. The films produce a **negative image**, so that the shadows of bones appear light. There is no reason, other than its being a waste of time and money, that the positive image could not be printed. Doctors examine the developed films on **light boxes**. With a broken bone, the problem is easy to see (*Figure 86*); looking for small cancers is not so easy.

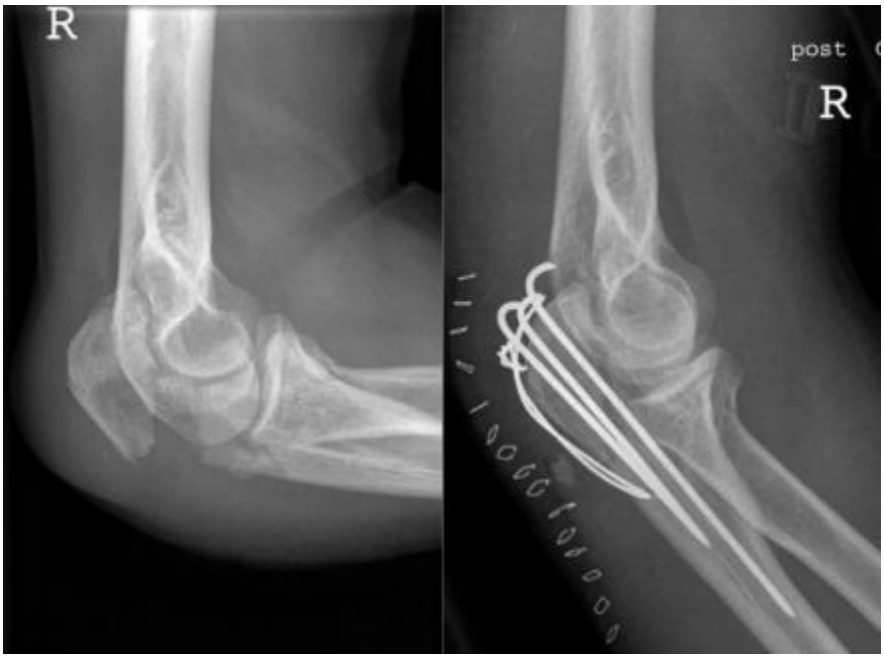


Figure 86 Radiograph of a broken elbow and its repair (Radiograph from Michael Müller-Hillebrand, Wikimedia Commons)

To reduce the exposure of a patient, the film is placed in an **image intensifier** (*Figure 87*). If you have had an X-ray in hospital, you will have seen these as the metal cases that contain the film. The intensifier screen is a layer of zinc sulphide, a **fluorescent** material, that glows (fluoresces) when exposed to X-rays. It absorbs the X-rays and retransmits them as visible light. The light then deposits the silver grains on the film as well as the X-ray photons. These devices can intensify the image by about 40 times, although the resolution is decreased a little. The best resolution is about 0.1 mm.

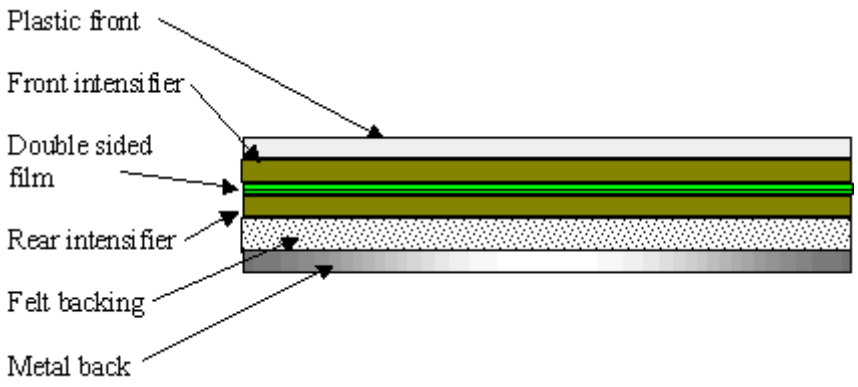


Figure 87 Image intensifier using photographic film

The use of a fluorescent screen (without a film) can allow doctors to view events in real time. This diagnostic method is called **fluoroscopy**. To get a decent image, though, you need quite a high intensity. In the old days machines with fluorescent screens were available as an amusement in shops or fun fairs. Nobody knew or cared about the risks then.

Image intensifier tubes can be used to avoid an increased dose of X-rays (*Figure 88*). The fluorescent screen is connected to a photocathode. Electrons are accelerated onto a second zinc sulphide screen, intensifying the original image by a factor of 1000.

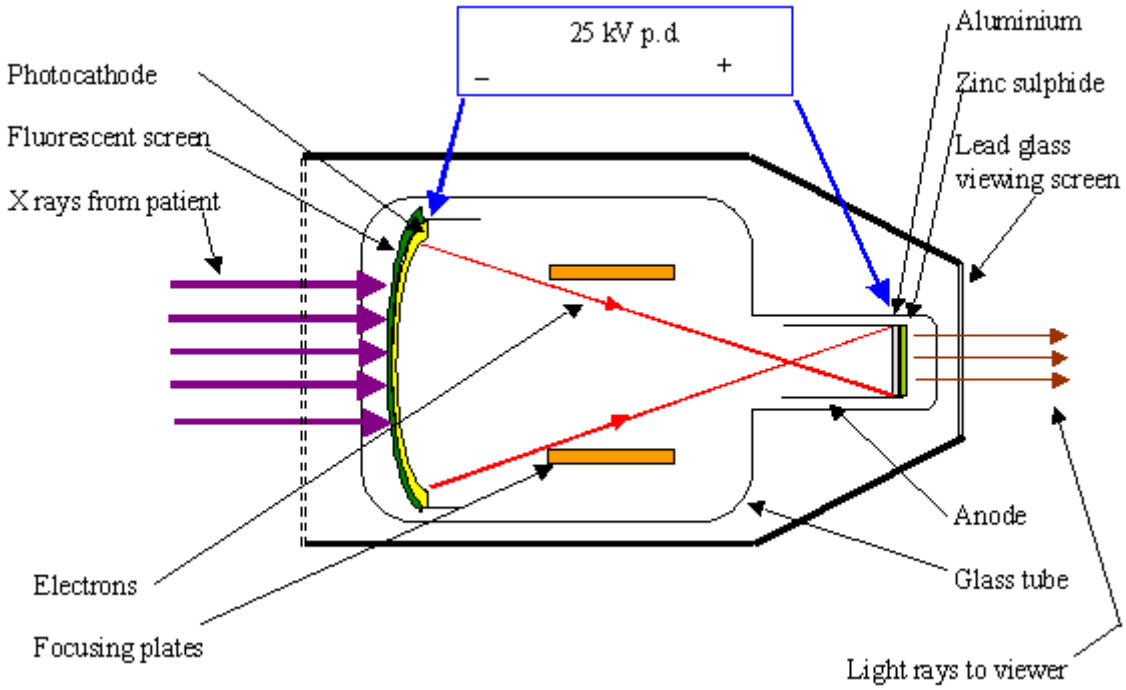


Figure 88 Image intensifier using a photocathode

Light from the second zinc sulphide screen passes to a TV camera for recording or direct viewing. Computer displays are now widely used.

X-ray shadows are clearest where there is greatest difference in density of the tissues. For example, bones are quite **opaque** to X rays. Soft tissues are slightly opaque while air is transparent. Lungs full of air show readily. The **contrast can** be increased by using a material that is opaque to X-rays. Studies of the function of the gastro-intestinal tract are carried out in real time, using the X-ray opaque material barium in the form of a **barium meal**. This shows up readily on X-rays (*Figure 89*).

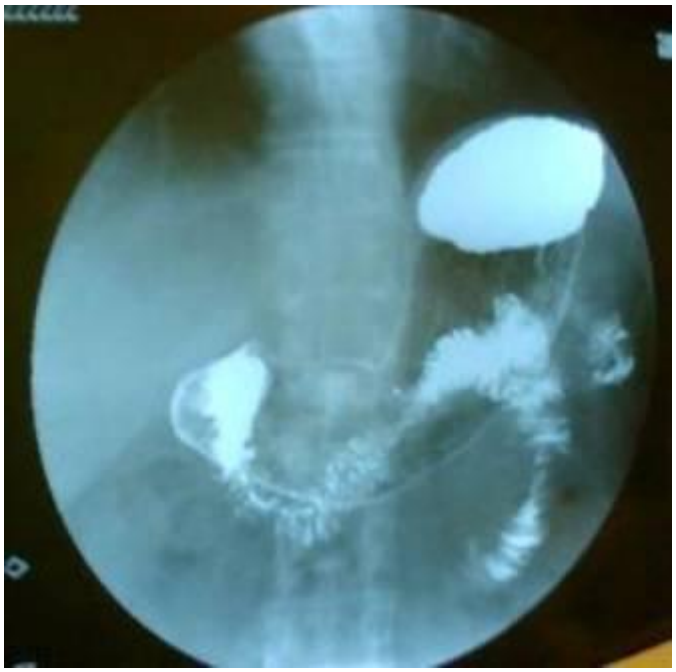


Figure 89 A barium meal

14B.078 Uses for X rays

X-rays are a common diagnostic tool. It is non-invasive, but there are risks due to the energetic radiation. As well as the normal shadow pictures, X-ray tomography makes images of cross sections of the whole body. This can be useful if there are a number of diseased sites in the body.

Energetic X-rays are used to treat cancer in a process called radiotherapy. The tumour is exposed to high energy X-rays and killed. However, there can be side effects. Also, the dose has to be worked out carefully. 10 percent less dose can leave a tumour unaffected, while ten percent more can damage the patient.

People **working** with X-rays have to take care as they could accumulate a high dose as they work:

- They wear a film badge to check the amount of radiation they get.
- They wear lead aprons while the machine is turned on.
- The machine is in an enclosed room, and the controls are in a separate room.
- Interlocks are arranged so that nobody can walk into the X-ray room while the machine is turned on. If that were to happen, the machine would be turned off immediately.

14B.079 CT Scan

CT stands for computed tomography. The CT-scanner uses X-rays to take pictures using a head that rotates around the patient's body. The imaging data are sent to computers that use powerful programs to produce an image that makes sense to the doctor. A picture of a CT scanner is shown below (*Figure 90*).

Figure 90 A CT scanner (Image by daveynin, Wikimedia Commons.)

As you can see, the machine is not as massive as the MRI scanner, nor is it as claustrophobic. Even so, the machine is complex and expensive. The picture below shows the machine with the covers removed (*Figure 91*):

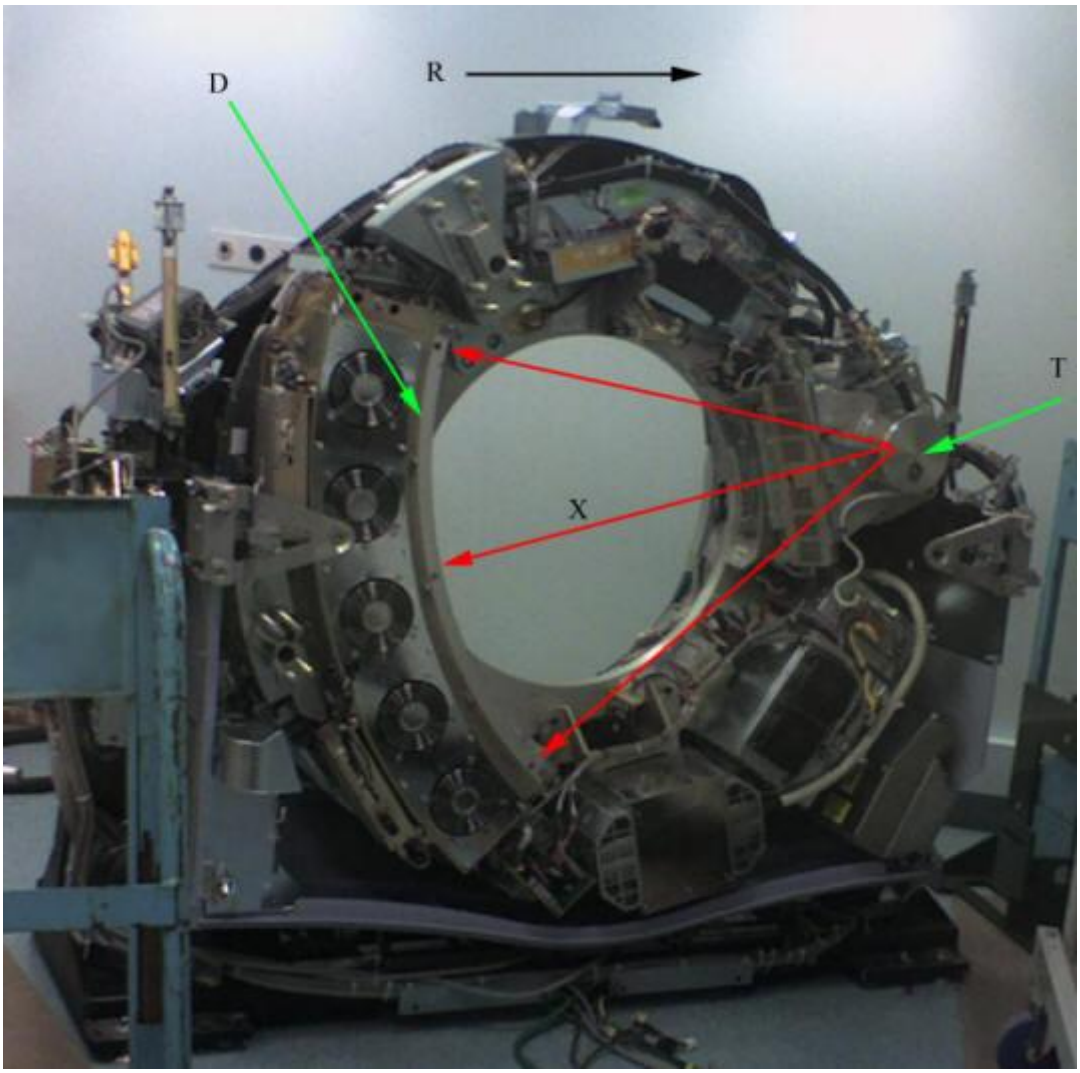


Figure 91 A CT scanner with the covers off (Image by ChumpusRex, Wikimedia Commons.)

The important parts are mounted in a circular **gantry**, so that they can rotate freely around the patient. The **X-ray** tube is marked ***T*** and produces the **X-rays** marked ***X***. They are picked up by an array of detectors, ***D***. The whole assembly rotates clockwise (***R***) on the gantry.

The X-ray tube produces X-rays that spread out as shown in the picture above, so that they are picked up by the array of detectors. Each detector will see a different view of the part of the body that is of interest. The output of the detectors is digitised, and these data are fed to the computer. *Figure 92* shows an example of how the raw data (left hand picture) are processed to produce a meaningful image of a jaw (right hand picture). A contrast-enhancing dye may be used to improve the quality of the images.

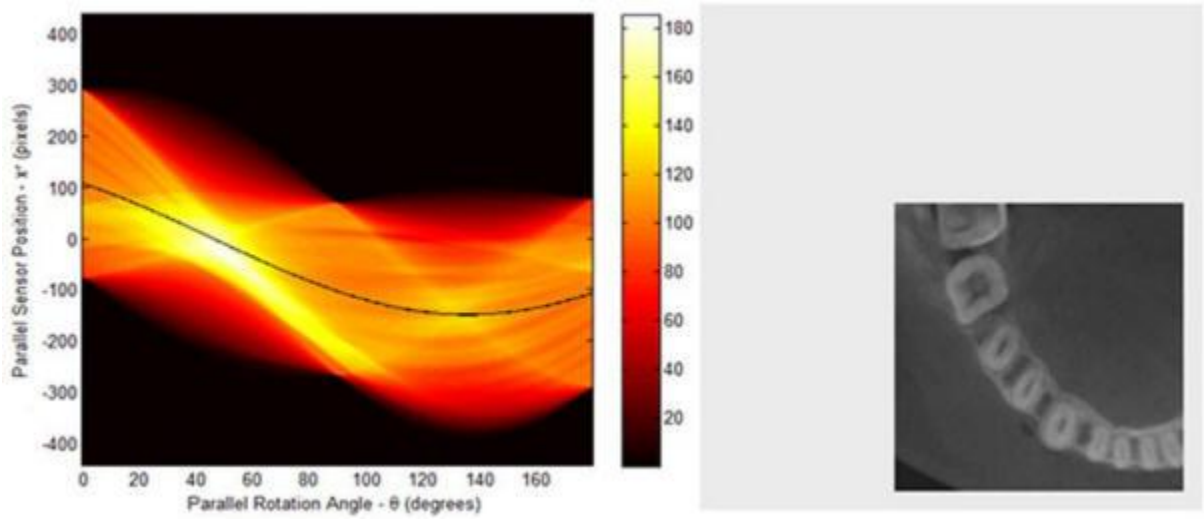


Figure 92 Raw data from a CT scan are processed by computer to make a meaningful image (Image by Kyungtaek Jun & Seokhwan Yoon, Wikimedia Commons)

The most common use for a CT scan is to produce images of virtual slices of the body (*Figure 93*). The ones shown are of the head:

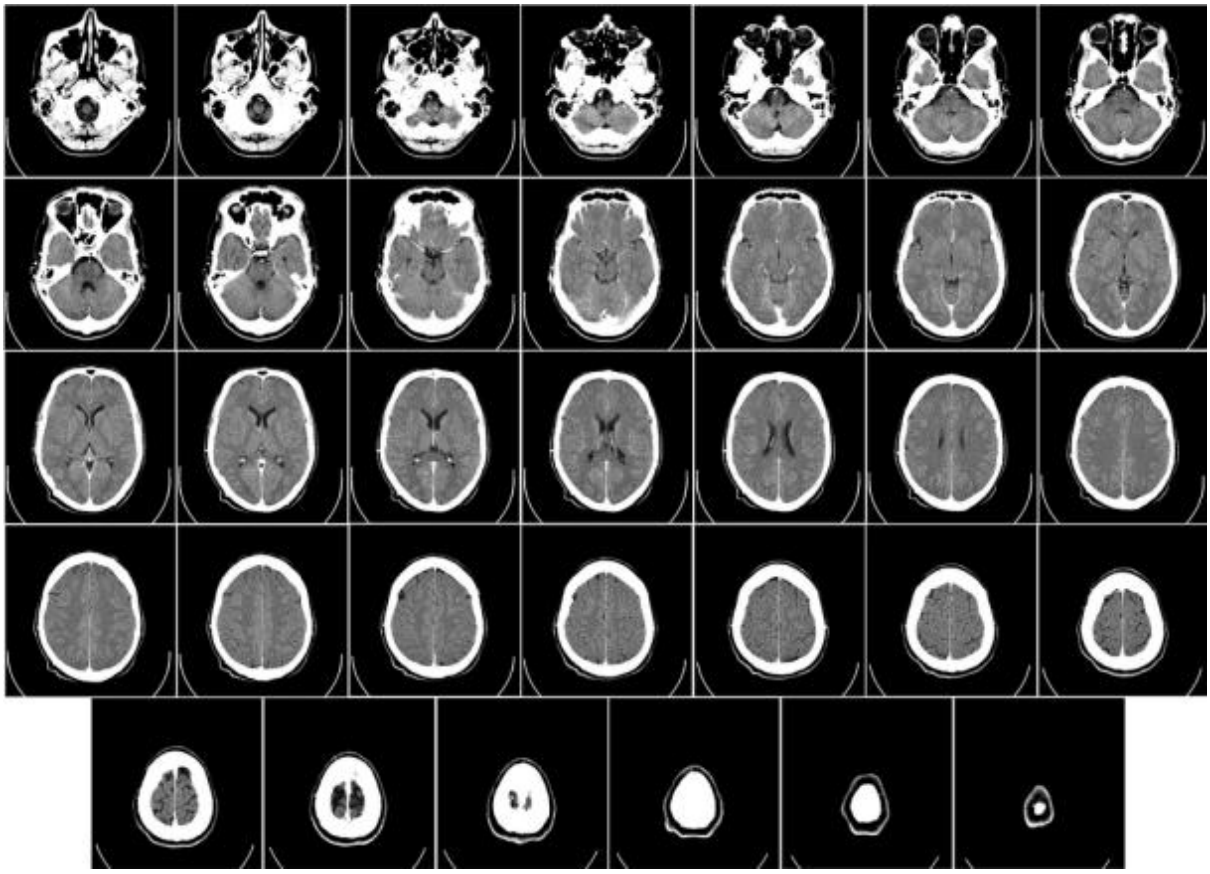


Figure 93 CT scan of the head (Image by Michael Häggström, Wikimedia Commons.)

You can see the brain and the eyes.

The data can be assembled to make three-dimensional images, or even animated.

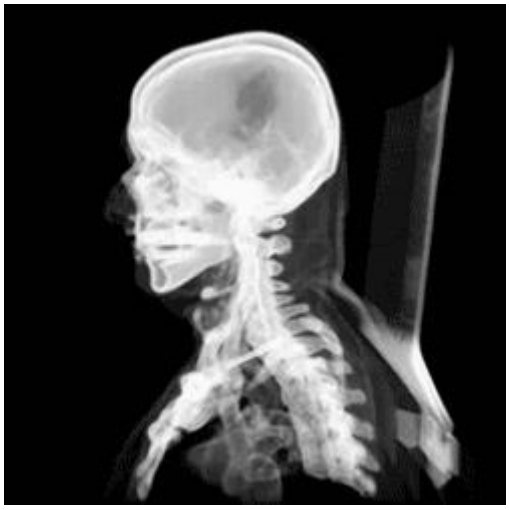


Figure 94 3-D imaged made by a CT scanner (Image by Arielinson, Wikimedia Commons)

The CT scan can be set up very quickly to allow the doctors to assess an emergency patient's condition very quickly, and to help them to decide how the patient is to be treated. The specific **advantages** are:

- Images can be rapidly acquired.
- The information gleaned from these images is clear and specific.
- The whole body can be scanned quickly.
- There is no risk from implants, as there is in MRI scans.
- The patient does not have to remain dead-still.
- Biopsies (an invasive procedure) can be avoided.

There are **disadvantages**:

- X-rays are used, which can cause mutations, which in turn may lead to cancerous growths.
- Children should not be exposed to radiation.
- The contrast enhancing dye may cause adverse reactions in some cases.
- Pregnant women should not be investigated using CT scans except in an emergency.
- Ovaries in women are more sensitive to damage than testicles in men.

14B.0710 Making 3-D Images (Extension only)

The CT scan uses very powerful computer software to make its images. It makes plane images of individual layers and stacks them on top of each other like this. The first layer is shown below (*Figure 95*):

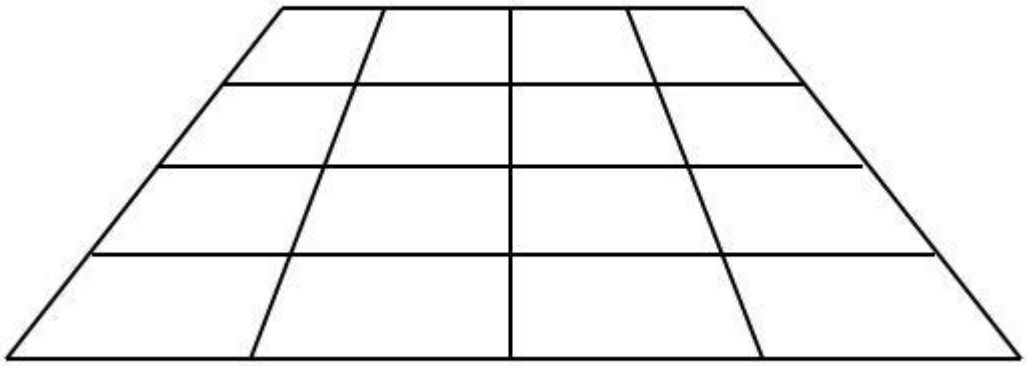


Figure 95 Making an individual layer.

The next layer is placed above the first layer like this (*Figure 96*):

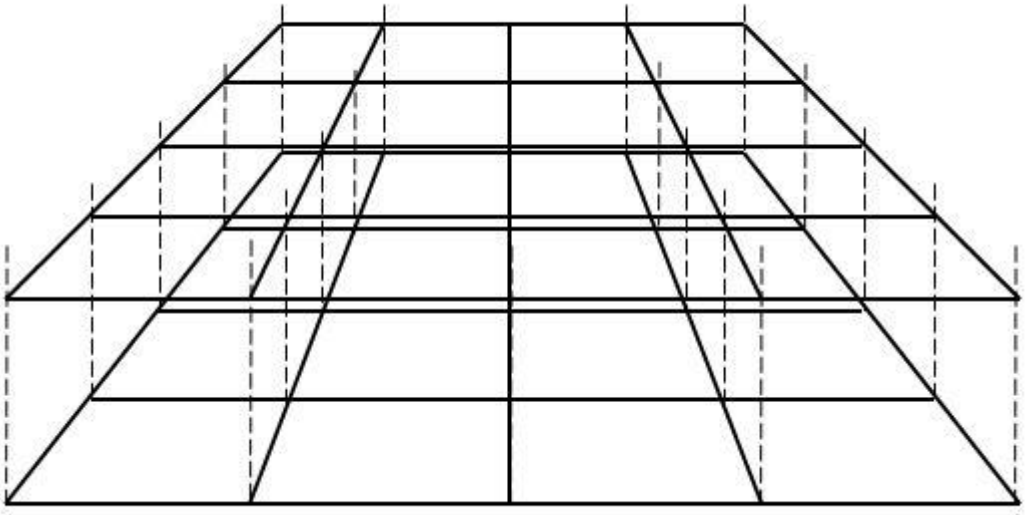


Figure 96 Adding a second layer

And the layers are built up like this (*Figure 97*):

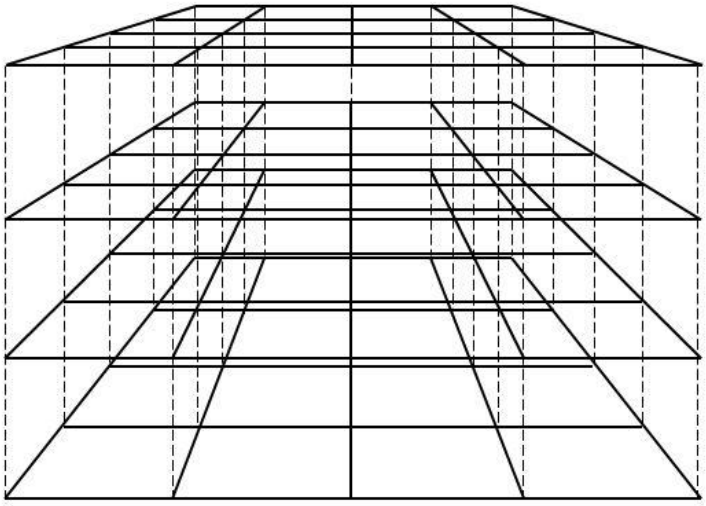


Figure 97 Adding more layers

You can see that there are 48 cubes in this grid. Suppose we now number the rows across, the layers and rows backwards. For simplicity we will say that they are on the x, y, and z axes (*Figure 98*).

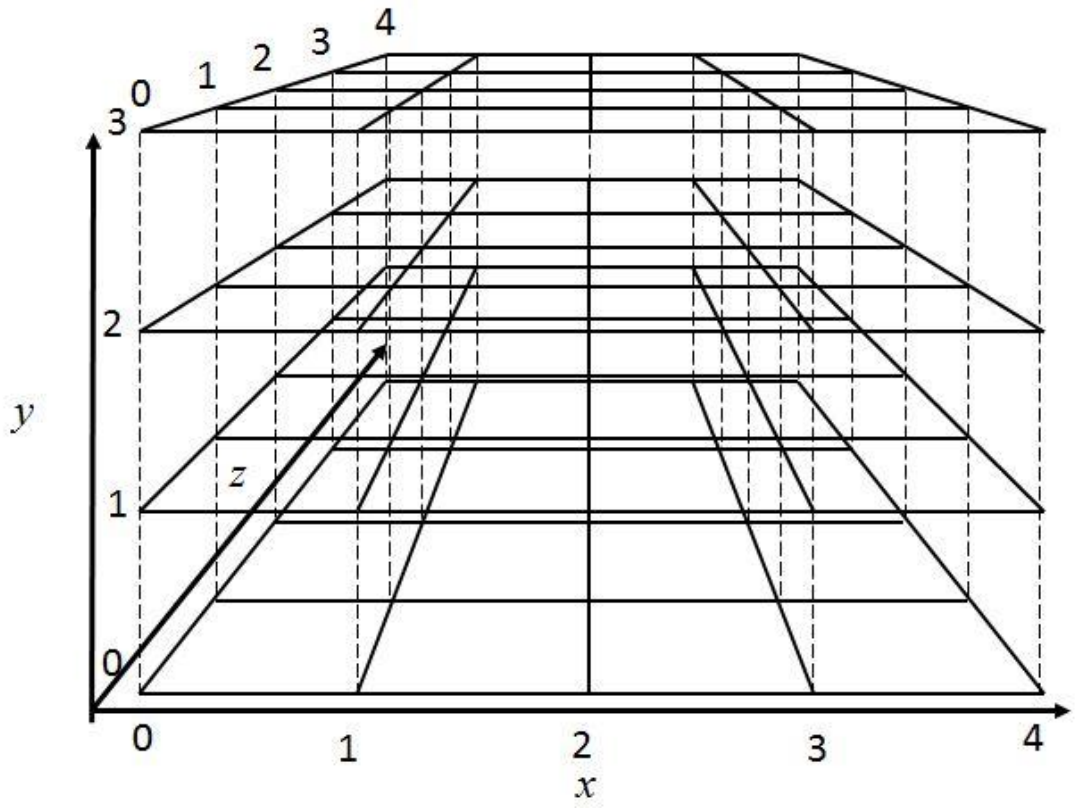


Figure 98 Numbering the cubes

The computer has a plan of all the cubes and their coordinates. Look at the cube below (*Figure 99*):

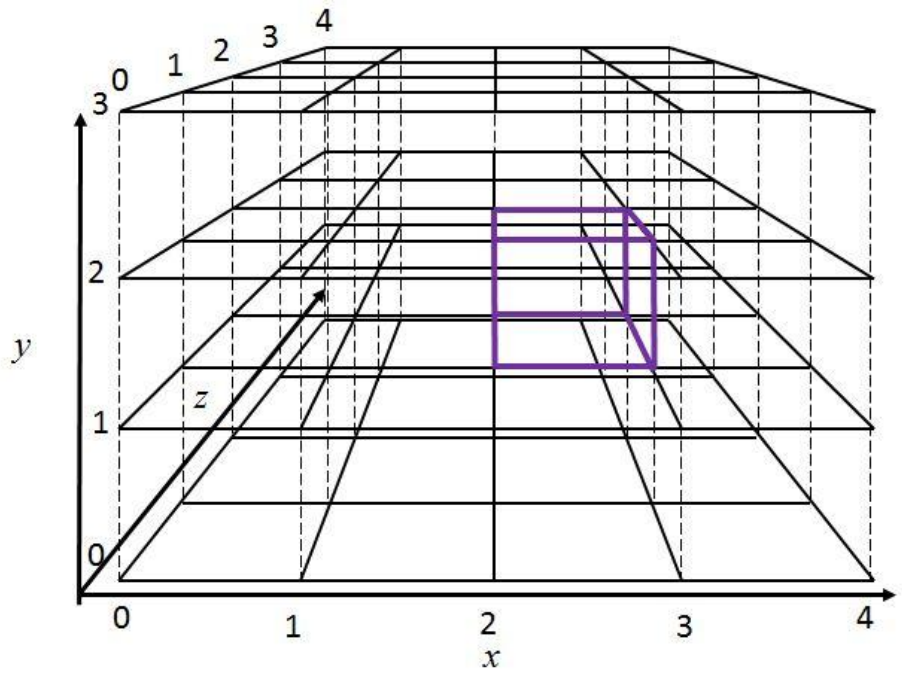


Figure 99 Positioning a particular cube (a voxel)

The x, y, z coordinates to describe this cube require 8 values to establish its position in space.

x	y	z
2	1	1
3	1	1
2	2	1
3	2	1
2	1	2
3	1	2
2	2	2
3	2	2

Each corner has a value to describe its position.

This cube is called a **voxel**. The voxel is a 3-dimensional equivalent of a pixel. Voxel comes from volume element, just as pixel is derived from picture element. The 8-voxel is a cube, as it has 8 vertices. Then we add another voxel to form a shape consisting of two cubes:

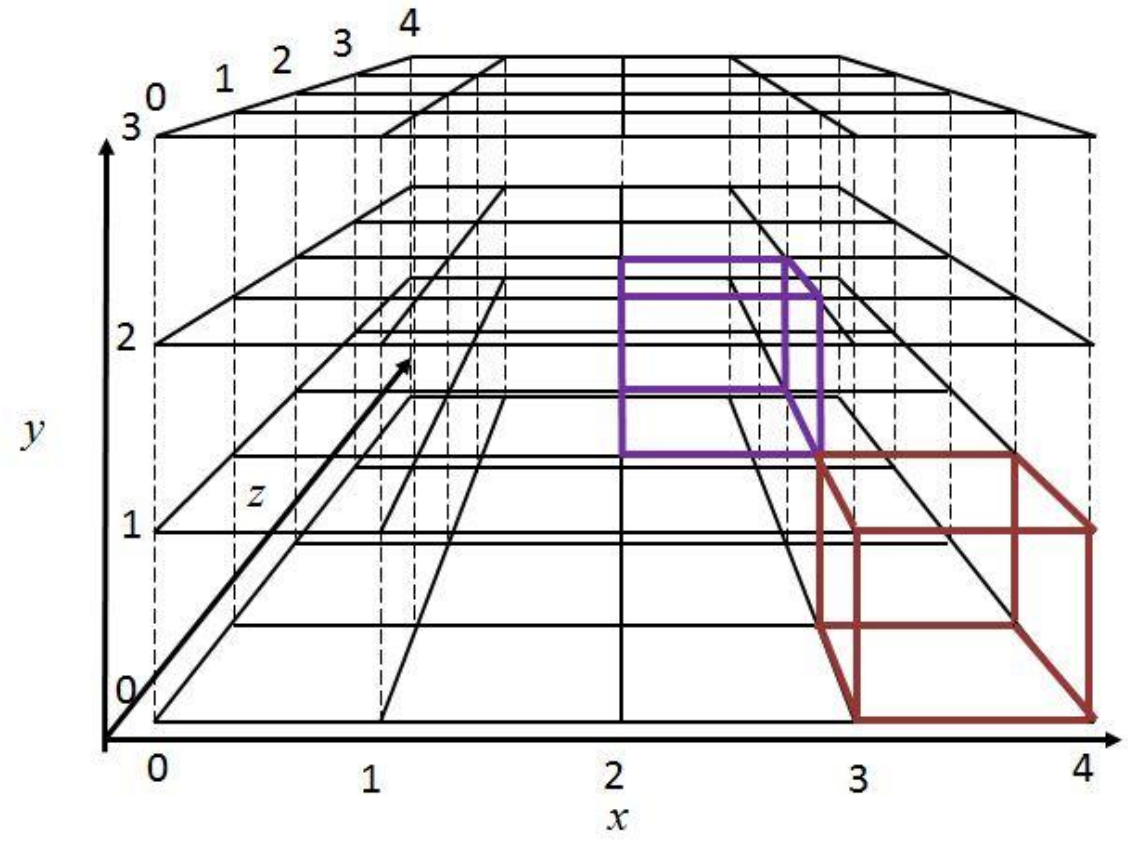


Figure 100 Adding a second voxel

We would need another 8 lines of coordinates to give the outlines of the whole shape. You can see it's starts to get complicated, even with a grid that can make a shape that has a maximum of 48 cubes.

In a CT scanner, the grid has many thousands of values in the x, y, and z directions, leading to a huge data table. If there were 10^4 values on each dimension, there would have to be 10^{12} spaces in the table. Each value would have to be a 16-bit word, leading to a data storage of 6.6×10^{16} bits or 8.1×10^{15} bytes (about 8000 TB). Clearly the data would need to be compressed by appropriate software in order to manage such huge files.

The data are manipulated by powerful software to enable the doctors to look at 3-dimensional pictures of the area of interest. The picture below shows a macromolecule

of which the surface has been smoothed to make it look less as if it were made of cubes (*Figure 101*).

Figure 101 A macromolecule of which the surface has been smoothed out (Image from Wikimedia Commons)

Questions

Tutorial 14B.07

14B.07.1

Rearrange *Equation 31* to make λ the subject and, by substituting appropriate values, use it to give you a rule that will give you the wavelength if you know the voltage.

14B.07.2

What is the minimum wavelength gained from electrons accelerated by a p.d. of 50 000 V?

14B.07.3

An X-ray tube is operated at a peak voltage of 100 kV, and the beam current is 40 mA.

(a) What is the power of the machine?

(b) How many electrons reach the machine every second?

(c) How many photons are released every second?

(d) What is the maximum energy of each photon? What is its wavelength?

[Data: $e = 1.6 \times 10^{-19}$ C; $h = 6.63 \times 10^{-34}$ J s; $c = 3.0 \times 10^{8}$ m s^{-1}]

14B.07.4

An X-ray machine is accelerating electrons through a p.d. of 200 kV. The anode current is 25 mA. The target is a block of a very heavy metal mass 1.0 kg, and specific heat capacity 300 J kg K^{-1} and melting point 3000 K. The machine is at 300 K when it is turned on. 10 seconds after the machine has been turned on the cooling fails. The machine continues to run for 3 minutes to sterilise some instruments and the operator has gone off somewhere. What do you think the operator will come back to? Explain your answer.

14B.07.5

Why can lung disease not be detected by ultrasound?

14B.07.6

What is the mass attenuation coefficient of copper? Density of copper is 8930 kg m^{-3}.

14B.07.7

The half value thickness of aluminium is 3.2 mm. What is the total linear attenuation coefficient?

14B.07.8

An X ray tube operates at a voltage of 80 kV and a tube current of 50 mA. As the X-rays leave the tube the area of the beam is 10 mm^2. The efficiency of the tube at producing X rays is 1 %. What is the intensity of the X-rays as they leave the tube?

14B.07.9

Assume that the intensity you worked out in Question 8 occurred 0.01 m away from the anode. What is the intensity 1.5 m away from the X-ray tube? [Hint: it's in air].

14B.07.10

What is the intensity of the beam behind a sheet of aluminium 5 mm thick placed 1.5 m away from the tube?

14B.07.11

Why should an observer not view the viewing screen directly?

Tutorial 14B.08 Imaging with Ionising Radiation	
AQA Syllabus	
Contents	
14B.081 Imaging Techniques	14B.082 Iodine 131
14B.083 Technetium-99m	14B.084 Indium-111
14B.085 PET Scanner	14B.086 Half Life
14B.087 Gamma Camera	14B.088 High Energy X-rays
14B.089 Radioactive Implants	14B.0810 Imaging Comparisons
14B.0811 Dosimetry (Welsh Board and Eduqas only)	

Before you attempt this tutorial, you may find it helpful to revise the tutorials on Topic 12 (Nuclear Physics) especially Tutorial 12.038 which covers metastable nuclei. You should be aware of the concept of half-life (Tutorial 12.05). A good understanding of all of these will be assumed throughout.

14B.081 Imaging Techniques

Many imaging techniques use the idea of **tracers**. A radioisotope can be injected as a **salt**, or a radioactive isotope that is attached to:

- a **biological molecule**.
- a **pharmaceutical molecule** (a drug).
- or a **carrier molecule**.

The molecule is taken to a particular **target organ** (*Figure 102*). Then the doctors can see the way the way that molecule is processed by that organ.

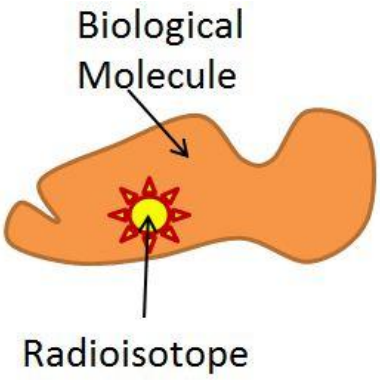


Figure 102 Molecule carrying a radioisotope

The radioisotope is a gamma emitter. An alpha emitter would not only be useless, but also highly dangerous.

A beta emitter is less ionising. In air, the range is less than a metre, and in body tissues it's typically a few millimetres. While such a range can be used for treatment of tumours, it is no good for imaging.

Gamma rays can pass out of the body easily, so these are used for imaging. We will look at the radioisotopes that are most commonly used.

14B.082 Iodine 131

Iodine is a component of **thyroxin** which made by the thyroid gland and is important in maintenance of the body's metabolism. The radioisotope is administered as radioactive potassium iodide, which is taken up by the thyroid gland.

Iodine-131 is a radioactive isotope of iodine, made in a research **reactor**. It is made from tellurium-130 (which is radioactive by beta minus decay with half-life of 8.2×10^{20} years) which absorbs a neutron:

$$^{130}_{52}\mathrm{Te} + ^{1}_{0}\mathrm{n} \rightarrow ^{131}_{52}\mathrm{Te}$$

..... *Equation 37*

This isotope of tellurium decays by beta minus decay:

$$^{131}_{52}\mathrm{Te} \rightarrow ^{131}_{53}\mathrm{I} + ^{0}_{-1}e^{-} + \bar{\nu}_{\varepsilon}$$

..... *Equation 38*

The half-life is about 25 minutes. (This contrasts with Te-128, which has a half-life of 2×10^{24} years, which is quite a long time, 10^{11} life times of the Universe.)

Iodine 131 is used widely for imaging. It is also used for **radiotherapy**, which is the treatment of cancerous tumours by radioactive emission. It has a half-life of 8.02 days (in SI units 6.93×10^5 s). The radioisotope decays to xenon with a decay energy of 970 keV by beta minus decay. 90 % of the iodine decays by beta-minus decay in two steps like this:

$$^{131}_{53}\mathrm{I} \rightarrow {}^{131}_{54}\mathrm{Xe}^{*} + \beta^{-} + \bar{\nu}_{\varepsilon} + 606\ \mathrm{keV}$$

.....*Equation 39*

The xenon nucleus (shown by Xe^{*}) is excited and almost immediately loses its energy as a gamma-ray photon:

$$^{131}_{54}\mathrm{Xe}^{*} \rightarrow {}^{131}_{54}\mathrm{Xe} + \gamma + 364\ \mathrm{keV}$$

..... *Equation 40*

The gamma photon has an energy of 364 keV.

For the other 10 %, the decay pattern is like this:

$$^{131}_{53}\mathrm{I} \rightarrow {}^{131}_{54}\mathrm{Xe}^{*} + \beta^{-} + \bar{\nu}_{\varepsilon} + 334\ \mathrm{keV}$$

.....*Equation 41*

The gamma photon has an energy of 636 keV:

$$^{131}_{54}\mathrm{Xe}^{*} \rightarrow {}^{131}_{54}\mathrm{Xe} + \gamma + 636\ \mathrm{keV}$$

.....*Equation 42*

The two pathways are shown in the diagram below (*Figure 103*):

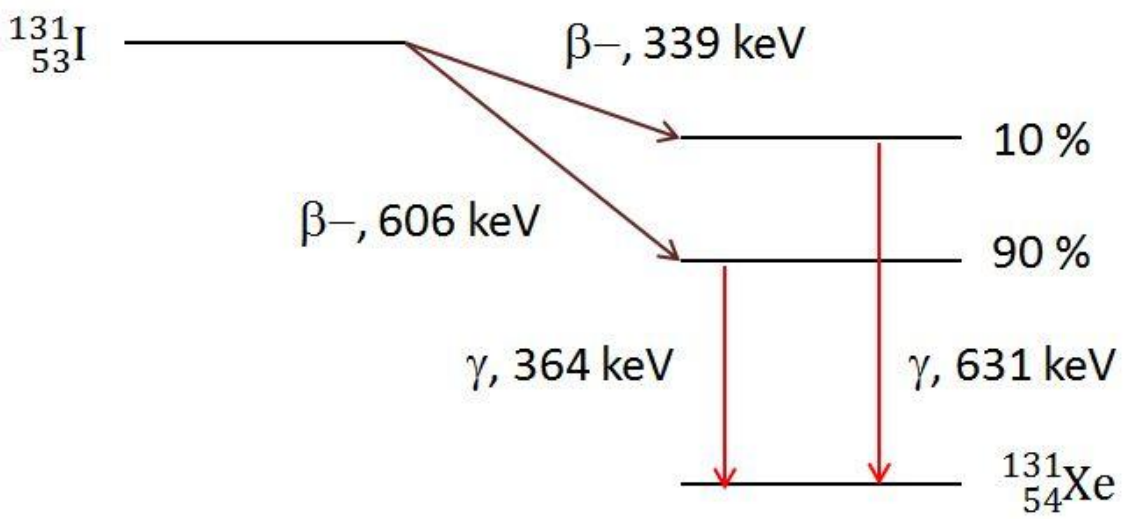


Figure 103 Decay of Iodine-131

The beta particles have a tissue penetration of about 2 mm. The image comes from the gamma rays. The picture below shows the accumulation of I-131 in the thyroid (*Figure 104*):

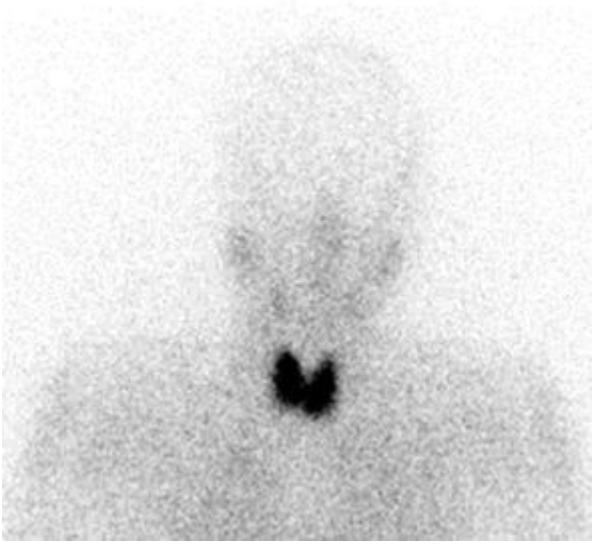


Figure 104 Iodine-131 in the thyroid (Image by Drahreg01, Wikimedia Commons)

Iodine-131 is also a fall-out product of nuclear fission. Uncontrolled exposure can lead to cancers of the thyroid. Iodine-131 is also used as an industrial tracer.

Patients who are treated with I-131 have a high radioactive burden for quite a time. After a month the radioactivity is about 7 % of the original. Therefore, the patients may have to keep away from their children or grandchildren for several days. (My late mother had some iodine treatment, and she was told that she should not see any of her grandchildren for about two weeks at least.)

14B.083 Technetium-99m

This is one of the most commonly used radioisotopes used in medicine nowadays. Almost 85 % of diagnoses that use radioactive imaging use Technetium as a radioactive tracer. There are a large number of pharmaceutical molecules that act as a vehicle. Tissues that are investigated in this way include:

- Brain.
- Myocardium (muscles of the heart).
- Thyroid.
- Lungs.
- Liver.
- Kidneys.
- Skeleton.

Technetium-99 m is made by bombarding moybdenum-98 with neutrons to give Mo-99:

$$^{98}_{42}\mathrm{Mo} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{99}_{42}\mathrm{Mo}$$

..... *Equation 43*

This decays to technetium-99 by beta minus emission:

$$^{99}_{42}\mathrm{Mo} \rightarrow {}^{99\mathrm{m}}_{43}\mathrm{Tc} + {}^{-1}_{0}e^{-} + \overline{\nu}_{\varepsilon}$$

..... *Equation 44*

The half-life for the decay is 66 hours (2.38×10^5 s).

The technetium nucleus is **metastable**, which means that its remains in an excited state for an extended period of time. Most excited nuclei lose their excess energy almost immediately (about 1.0×10^{16} s). The half-life of the metastable state is 6.03 hours (21700

s). As the energy level of the nucleus falls, a gamma photon of energy 140 keV is emitted. The low energy technetium decays to ruthenium by beta minus decay, with a half-life of 211 000 years (6.65×10^{12} s). The energy loss events of the metastable nucleus form an exponential decay, just like any other radioactive decay.

The energy of the gamma photons is low, so the radiation is less ionising, so it safer to use than other radionuclides. Additionally, the half-life of the metastable state is short. The low energy technetium has a low rate of decay and is excreted from the body in the urine.

Technetium has to be prepared on site for immediate use, as the metastable state has a half-life of just 6 hours. It is supplied as Mo-99 in the form of molybdate ions (MoO_4^{2-}), and placed into a technetium generator, sometimes light-heartedly called a **moly-cow**. Aluminium oxide in a column bonds with the molybdenum but does not bond with the technetium. Saline is passed down the column in a process called **column chromatography**. The technetium in the form of pertechnetate ions (TcO_4^-) is flushed out with a saline solution. The process is sometimes called a moly-milk. The idea is shown in this simplified diagram (*Figure 105*):

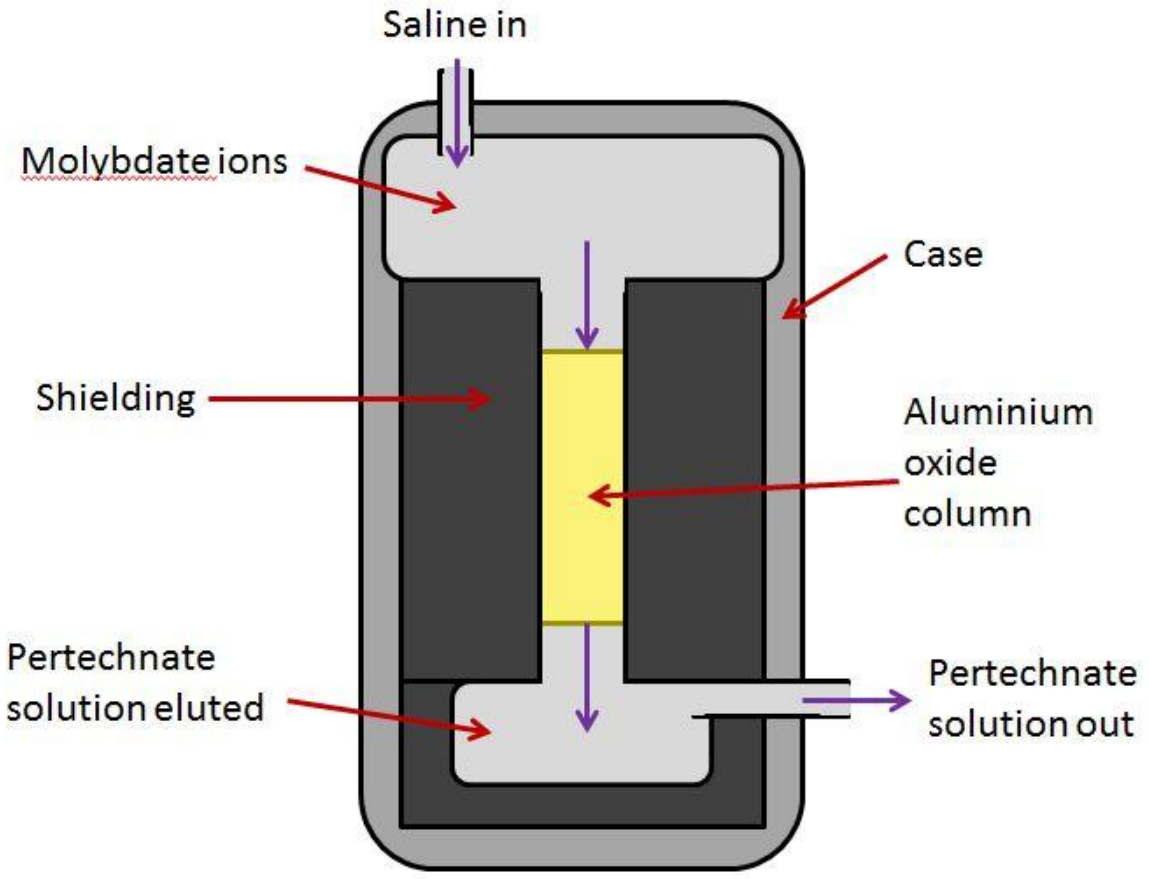


Figure 105 A moly-cow (technetium generator)

The pertechnetate ions may then be bonded to a pharmaceutical. Then they are injected into the patient. Some technetium generators are shown below (*Figure 106*):

Figure 106 A technetium generator in a hospital (Image by Kieran Maher, Wikimedia Commons)

The technetium generator and the syringes used for injection are shielded to reduce exposure to radiation. Since the molybdenum-99 has a half-life of 66 hours, hospitals need to buy the generators on a regular and staggered basis.

14B.084 Indium-111

Indium is produced by bombarding cadmium (Cd) nuclei with protons in a **cyclotron**. The event is shown in the equation below:

$$^{110}_{48}\mathrm{Cd} + {}^{1}_{1}\mathrm{p} \rightarrow {}^{111}_{49}\mathrm{In}$$

..... *Equation 45*

The product formed does NOT decay by beta minus decay, but by **electron capture** to make Cd-111:

$$^{111}_{49}\mathrm{In} + {}^{0}_{-1}e^{-} \rightarrow {}^{111}_{48}\mathrm{Cd} + \nu_{\varepsilon}$$

..... *Equation 46*

The daughter cadmium nucleus is left in an excited state. It loses the excess energy by emitting a gamma photon. There are two energies for the gamma photons, 171.3 keV and 245.4 keV. The half-life for the electron capture is 2.80 days (2.42×10^5 s).

The vehicles used to carry indium-111 include:

- antibodies.
- oxine, an organic compound that allows the indium to be used as a tracer for cell components of blood.
- peptides and proteins that are taken up by tumours.

14B.085 PET Scanner

PET stands for **Positron Emission Tomography**. A PET scanner is shown in the picture below (*Figure 107*):

Figure 107 A PET scanner (Image: Jens Maus - Wikimedia Commons)

A biologically active molecule acts as a vehicle for a positron emitter and is injected into the patient. The most widely used molecule is fluorodeoxyglucose, which is taken up by organs like the brain.

The tracer used is **fluorine-18**, which has 9 protons and 9 neutrons. It is made by bombarding water that carries the stable isotope oxygen-18 with high speed protons (18 MeV) from a cyclotron.

$$^{18}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{p} \rightarrow {}^{18}_{9}\mathrm{F} + {}^{1}_{0}\mathrm{n}$$

..... *Equation 47*

(In all the sources I have seen, they simply say that oxygen-18 takes in a proton to form a nucleus of fluorine-18. No mention of the neutron is made, but it is needed to make the equation balanced.)

Fluorine-18 is not stable and decays by beta plus emission to stable oxygen-18. This is because there are too few neutrons for stability. The half-life of the beta plus decay is about 110 minutes (6586 s). The energy of the positrons is 633.5 keV. This type of decay accounts for 97 %. 3 % of the decay is by electron capture.

The positrons travel about 1 mm before meeting an electron. A **positronium** particle is formed (a positron and electron).

Notice that the gamma photons move at an angle to the path of the positron and the electron, rather than perpendicularly (as seen in Topic 2, Tutorial 2.055). This is because the positron is moving fast, while the electron will be moving much slower. Momentum has to be conserved.

The gamma photons are picked up in an array of sensors that are mounted in a circular ring about the patient (*Figure 108*).

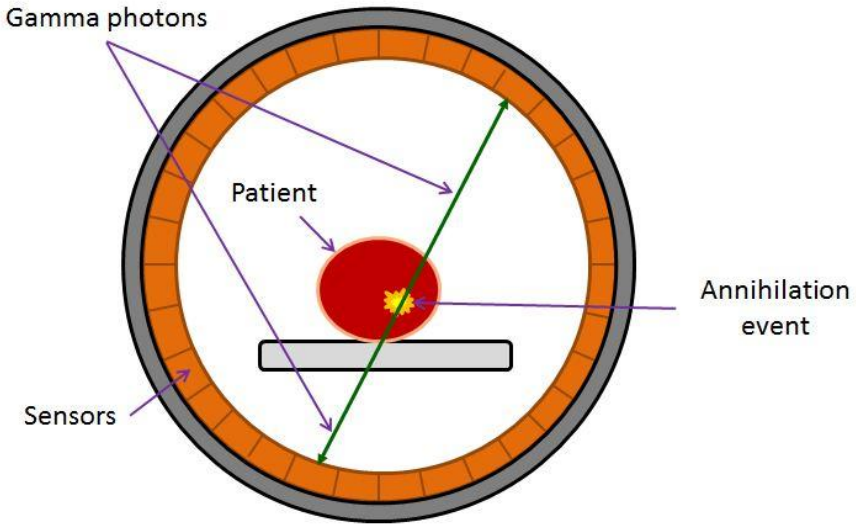


Figure 108 PET scanner sensors

The gamma photons move off in a straight line in opposite directions. Each pair strikes sensors around the sensor array. When each pair of sensors is activated, data are sent to the computer. The combination of sensors activated enables the computer to locate the source with precision. The PET scan does not make an image that is particularly easy to interpret on its own. The resolution is not good. At best it's about 0.5 mm, but can be as much as 6 mm. Therefore, the PET scan is often used in combination with a CT scan to produce high quality images to allow doctors to determine the shape and location of the tumour. The picture below (*Figure* 109) shows a 3-dimensional image from a PET and CT Scan.

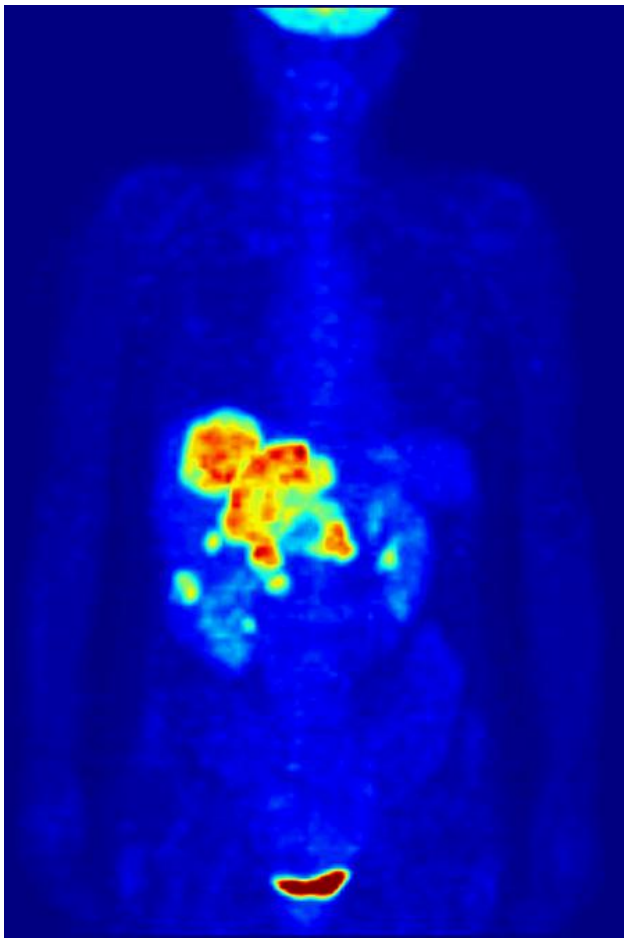


Figure 109 A 3-D image from a PET and CT scan

In this case, the tracer has collected in the brain, kidneys, bladder, and liver. Also, there are faint images of tumours in the abdominal cavity. This shows that **metastasis** has occurred, which means that the cancerous cells have broken away from the original tumour and spread about the body.

14B.086 Half Life

In Topic 12, Tutorial 12.05 we came across the equation for exponential decay:

$$N = N_0 e^{-\lambda t}$$

..... *Equation 48*

[N – no of nuclei; N_0 – original number of nuclei; e – exponential number, 2.718...; λ - decay constant (s^{-1}); t – time (s)]

We often use the activity rather than the number of nuclei, so the equation becomes:

$$A = A_0 e^{-\lambda t}$$

..... *Equation 49*

[A – activity (Bq); A_0 – original activity (Bq); e – exponential number, 2.718...; λ - decay constant (s^{-1}); t – time (s)]

In reality, the patient will not be emitting that number of gamma photons per second. This is because the body is eliminating the radiopharmaceutical through normal methods of waste disposal (i.e. going to the lavatory). Therefore, there are two factors in the **effective half-life** of a radiopharmaceutical:

- The **physical half-life**, which is a property of the radionuclide itself and cannot be changed at all.
- The **biological half-life**, the time in the body.

There are a number of factors that determine the length of the **biological half-life**. For example, if the tracer is targeted at bones, it can hang around there for quite some time. Other tracers are eliminated from the body quite quickly through the kidneys. The biological half-life is defined as:

The time taken for the body to eliminate half of the remaining radionuclide by natural means.

The natural means include excretion through the kidneys in urine, sweating and the body's metabolism. Sometimes it can be a few minutes, much shorter than the radio nuclide's half-life. Sometimes the physical and biological half-lives are about the same. Sometimes the biological half-life is quite a bit longer.

As well as the physical decay constant of the radionuclide, there is the biological decay constant. So, we can write:

Effective decay constant (s^{-1}) = physical decay constant (s^{-1}) + biological decay constant (s^{-1})

In Physics code, we write:

$$\lambda_E = \lambda_P + \lambda_B$$ ..... *Equation 50*

Since:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$ .....*Equation 51*

We can rewrite the equation as:

$$\frac{1}{t_{\frac{1}{2}E}} = \frac{1}{t_{\frac{1}{2}P}} + \frac{1}{t_{\frac{1}{2}B}}$$ ..... *Equation 52*

Note that doctors often measure the half-life in days, not seconds, as this is more meaningful to the patients. There are 86400 seconds in a day.

Worked example

Phosphorus-32 has a physical half-life of 14.3 days, and a biological half-life of 1155 days. What is the effective half-life?

Answer

$$t_E^{-1} = t_P^{-1} + t_B^{-1} = 14.3\ d^{-1} + 1155\ d^{-1} = 0.0708\ d^{-1}$$

$$t_E = \mathbf{14.1\ days}$$

Remember that you must add the **reciprocals** of the half-lives.

In the case of phosphorus-32, the tracer will have decayed before it is eliminated from the body.

14B.087 Gamma Camera

Some crystals can emit flashes of light (or **scintillate**) when exposed to gamma radiation. The **gamma camera** uses this property. It is constructed like this (*Figure 110*):

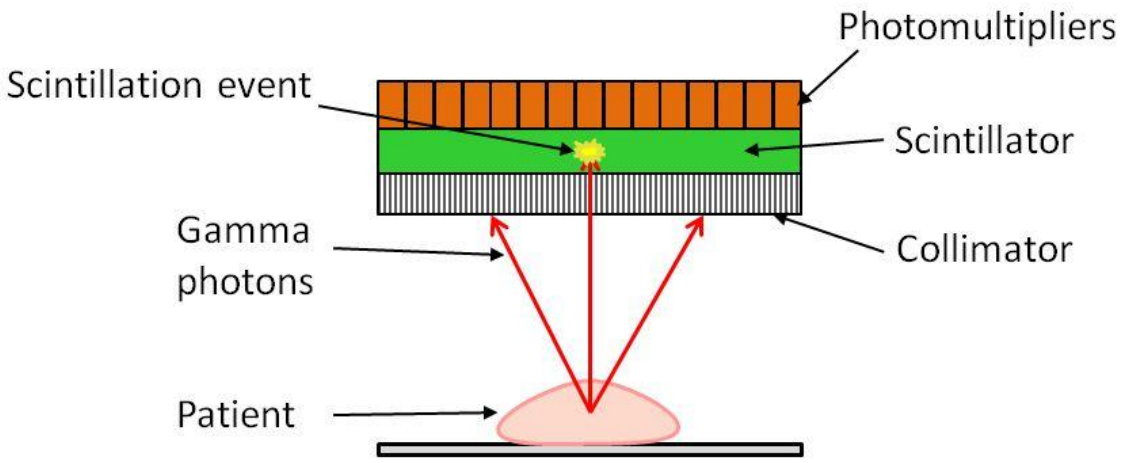


Figure 110 A gamma camera

The gamma rays from the patient spread out radially. In the diagram above, they are shown coming from one source. In reality the gamma rays would come from several sources. This could cause a very unclear image, with several scintillation events from each source. Therefore, a **collimator** is used. A collimator is a set of narrow tubes made from lead that selects only the photons that are coming up vertically. If the photons come at an angle, they are absorbed by the lead; the idea is shown below (*Figure 111*):

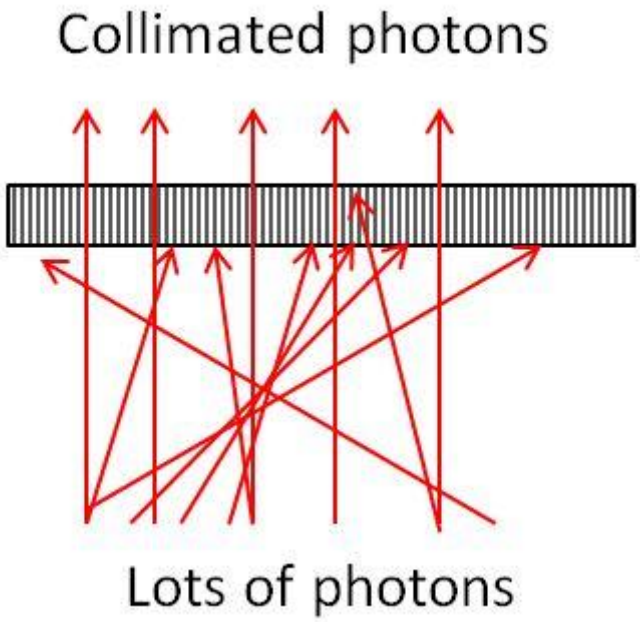


Figure 111 Action of a collimator

There are far fewer photons that strike the scintillator than there would otherwise. However, gamma rays (and X-rays) cannot be refracted by materials, so it's not possible to focus them with a lens.

Once the photons pass the collimator, they arrive at the **scintillator**, often a crystal of **sodium iodide**. Electrons are excited by the gamma photon and lose the extra energy as a visible light photon. This is picked up by the photomultiplier to be sent out as a signal to the computer (*Figure 112*).

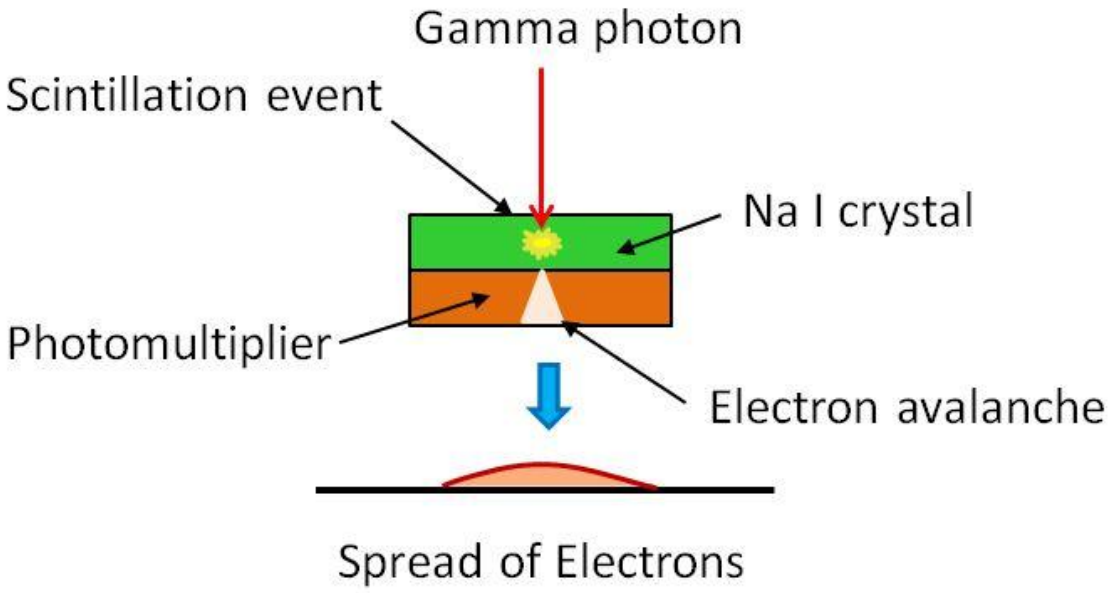


Figure 112 Detection of scintillation events by photomultipliers

This is an **indirect** process. The gamma photons have to set off a scintillation to be observed. The electron avalanche spreads out across the whole photomultiplier, which reduces the resolution. You can see how the electrons are spread out, which results in a lower resolution. The resolution of a gamma camera of this kind is about 10 mm.

One problem with sodium iodide is that it is **hygroscopic** (not hydroscopic) which means that it absorbs humidity from the air. Eventually they dissolve in the water they absorb, a process called **deliquescence**. So, the sodium iodide crystal has to be kept away from the air.

More recently the resolution can be improved by direct detection of the gamma photons by semiconductors, which can be made very small. The picture below show the idea (*Figure 113*):

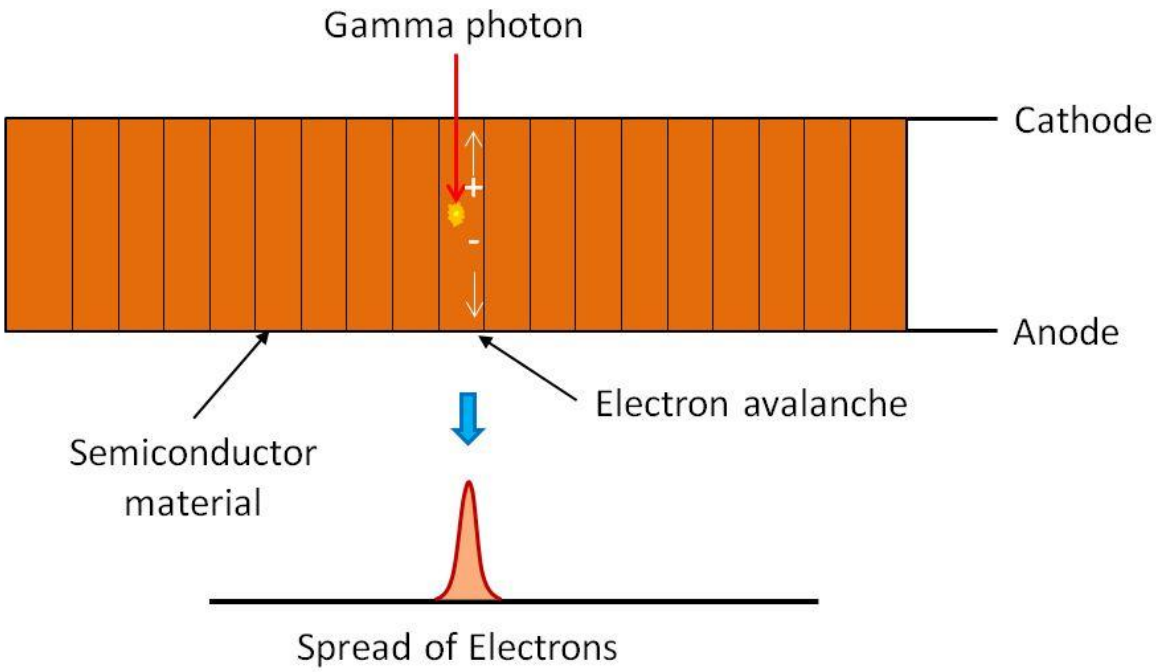


Figure 113 Detection of scintillation events by solid state photomultipliers

The avalanche of electrons is large and in a narrower spread, leading to better resolution. This type of gamma camera has a resolution is about 4 mm (which isn't that brilliant).

14B.088 High Energy X-rays

These are often referred to as **hard X-rays**. The X-ray photons carry an energy of between 100 keV to 1000 MeV. Their energies are comparable to gamma rays. (This is consistent with the idea of the electromagnetic spectrum being continuous. Soft X-rays can be considered to be hard UV.) They can be used to penetrate deep into matter, which makes them suitable for **radiotherapy**. See *Figure 114*.

Figure 114 A large X-ray radiotherapy machine (Image by Michael Goodyear, Wikimedia Commons)

In a diagnostic machine, the electrons are accelerated by a large voltage between a filament (like a light bulb) at 0 volts and a target at the anode. The potential difference is 50 kV.

The **radiotherapy machine** needs a much higher energy, so it uses a **linear accelerator** (LINAC) to accelerate electrons to hit a heavy metal target. In a research machine, alpha particle can be used, but the radiotherapy machine uses electrons. The linear accelerator alternates at microwave frequencies (about 1×10^{10} Hz). The accelerated electrons are bent by magnetic fields to strike a heavy metal target, like tungsten. The process is inefficient, and only 1 % of the accelerated electrons actually produce an X-ray photon. The rest of the energy is lost as heat. The X-rays can shaped by collimators to form the shape of the cancer. Even so, most of the X-ray photons that are produced do not leave the head but are absorbed by the casing. This is because they go off in random directions. The proportion that leave the head to irradiate the patient is quite low, approximately 10 % of the photons produced.

The idea of the machine is to target high energy X-rays to destroy the DNA of the cancerous cells, which will kill them. The head is movable, so it can be made to rotate about the patient. A static head would cause considerable damage to healthy cells near to the tumour, which is undesirable. Therefore, the head is **rotated** so that while the cancerous cells are bombarded, the healthy cells get a much lower dose. While there is risk of secondary cancers to the healthy cells, they generally repair themselves to remain healthy. The cancerous cells cannot repair themselves, so they die.

The X-ray dose is released in short bursts so the collateral damage to healthy cells is reduced as low a level as possible. The machine head is then moved to another position, so that another short burst can be given.

In some machines, a linear accelerator is not used. **Laser light** is shone onto the target to release the X-rays. These use titanium in a matrix of aluminium oxide to produce intense red light. With a sufficient intensity (in the order of 1×10^{12} W m^{-2}), X-rays can be produced.

Gamma rays are also used in a similar way for radiotherapy. The source of these is usually cobalt-60. In this case, the head does not need a power source. (The use of gamma rays for radiotherapy is not on the AQA syllabus.)

14B.089 Radioactive Implants

Sometimes it is necessary to carry out **internal radiotherapy**, which involves placing a radioactive source next to a cancer. The cancers most commonly treated in this way include:

- Breast cancer.
- Cervical cancer.
- Prostate cancer (the prostate is a gland found in men that produces a white oily secretion that carries sperm cells).

Other cancers can be treated in this way as well.

The most common treatment using a **radioactive implant** is called **brachytherapy**. The implant is simply a capsule that carries a radionuclide (*Figure 115*). It is inserted by a surgical procedure to place the capsules around the cancerous tissue.

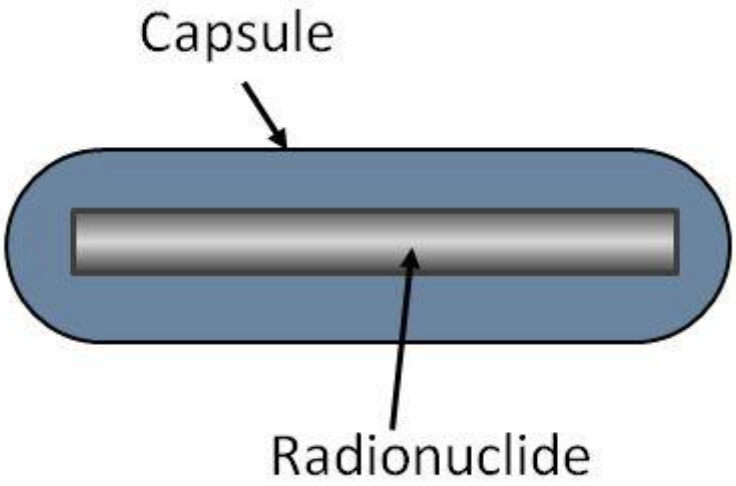


Figure 115 Capsule carrying a radionuclide

Sometimes bare wires made of radioactive material are used. They are called **radiotherapy wires**.

It is possible to use gamma rays or beta minus particles. Common beta minus emitters include:

Radionuclide	Half Life / d	Energy / eV
Iridium-192	73.8	3.8×10^5
Ruthenium-106	370	3.54×10^6

The beta particles penetrate into the cancerous cells and the ionising radiation does damage to the DNA. This is much more likely to kill cancerous cells than it is to kill healthy cells. A large dose can be applied to a small area, and does not require the use of large and expensive equipment to deliver the radiotherapy from outside. Often the patient can undergo the treatment at home, once the implants have been placed. The radiation risks to other members of the family are low, since the tissue penetration is only a few millimetres.

There can be short time side effects as with any procedure, such as bruising and discomfort in the implanted region. Longer term side effects could result from the irradiation of the area about the tumour, such as scarring, or even secondary cancers (which defeats the object of the treatment).

Liver cancer is treated using **selective internal radiation therapy** (SIRT). In this relatively new procedure, very tiny beads are injected into a blood vessel leading to the liver. They block the tiny blood vessels that feed the cancer. So, the cancerous cells are not only irradiated, but they are also starved of nutrition.

14B.0810 Imaging Comparisons

Initial tests to make a diagnosis are carried out by the **general practitioner** (GP). The kinds of tests carried out by the GP include:

- Visual tests of the mouth, ears, and nose.
- Stethoscope, an instrument that allows the doctor to listen to noises going on in the body.
- Measurements of temperature, and blood pressure.
- Taking blood and urine tests.

The imaging techniques described in this and previous tutorials are only carried out after the GP has exhausted the first line tests. They require specialist equipment that is expensive, and not always easy to interpret. The equipment is expensive to buy and to run. The personnel that use this equipment have to be well-trained. They are often called **radiographers**. The specialist doctors that make diagnoses from the radiology equipment are called **radiologists**. They work in the **radiology department** of a hospital.

We have seen how some diagnostic imaging equipment does not use ionising radiation, for example, the ultrasound scan and MRI scan. However, many scans use X-rays. The amount that these are used has to be carefully monitored, to ensure that the doses given do not put a patient at risk. A normal X-ray gives a dose that is a very small fraction of the overall safe dose that a patient can receive. The main principle that governs the use of radiation is "**as low as reasonably achievable**" (ALARA).

The use of radioactive tracers is more risky, and the radiology department would not wish to undertake such procedures unless the potential benefits out weight the risks. Clearly, if there is a condition that is life-threatening, or likely to be so, then use of diagnostic techniques that use ionising radiation is going to be essential. Doses of radiopharmaceuticals are very carefully calculated for the patient. Not all cancer

treatment is, of course, radiotherapy. **Chemotherapy** is often used. This can be effective but can have many unpleasant side-effects (they are highly toxic chemicals). They can make the patient feel very ill and lose their hair.

In the exam, you may be asked to compare **imaging techniques** for:

- Safety.
- Convenience.
- Resolution.

The question is most likely to be a "six-pointer", in which you need not only to describe the imaging techniques correctly, but also write coherently in good English.

14B.0811 Dosimetry (Welsh Board and Eduqas only)

The average activity of a radioactive material is defined as:

the number of disintegrations every second.

Over a **short period of time**, this statement can be summed up using the simple equation:

$$A = \frac{N}{t}$$

..... *Equation 53*

- A - activity (Bq).
- N - number of nuclei disintegrating.
- t - time (s)

The beta and gamma radiation will penetrate the body (alpha is stopped by skin). Even if the chance of an interaction was quite low, your answer to Question 14B.08.11 will show you that over a period of time, there would be a significant number of interactions, each of which could cause damage. So, we need to have a way of measuring the absorbed dose of radiation.

The **absorbed dose** is defined as:

energy absorbed per unit mass

In physics code, this is written:

$$D = \frac{E}{m}$$

..... *Equation 54*

- D - absorbed dose (units are Gray (Gy))
- E - energy (J);
- m - mass (kg).

1 Gy is the equivalent of 1 J kg^{-1}.

That is not the whole story, though. The risk of damage depends not only on the absorbed dose, but also on:

- The type of radiation (alpha, beta, gamma, or neutrons).
- The type of tissue that is exposed to the radiation. Some tissues are more sensitive to damage than others.

So, we give a **weighting factor** for each kind of radiation:

Radiation	*Weighting Factor W_r*
Alpha	20
Beta	1
Gamma	1
Slow neutrons	3

A slow neutron is also known as a thermal neutron. It travels at about 14 km s^{-1} which may sound fast but is quite slow as far as particles are concerned. The table shows that alpha particles are very damaging if they manage to penetrate the body.

The Weighting Factor is combined with the absorbed dose to give the **equivalent dose**, which is defined as:

the product between the weighting factor and the absorbed dose

It has the physics code, H, and the units are Sieverts (Sv). The equation is:

$$H = DW_r$$

..... *Equation 55*

The time of exposure needs to be taken into account as well. A dose of, say, 15 millisieverts (mSv) will have more impact on the body if it is received in 1 day than if it is received in a year. This is because the body has very good repair mechanisms. So, we need another quantity called the **equivalent dose rate**, which is the rate at which the equivalent dose is received. As an equation, this is written as:

$$\dot{H} = \frac{H}{t}$$

..... *Equation 56*

- H-dot - the equivalent dose rate (Sv s^{-1}).
- H - equivalent dose (Sv).
- t - time (s).

You don't have to use seconds for time. You can use hours, or minutes, as long as you are consistent. Use the time units given in the question.

Questions

Tutorial 14B.08

14B.08.1

Why is an alpha emitter no good for tracers?

14B.08.2

Calculate the speed at which an electron of energy 334 keV travels. Comment on your answer.

Electronic charge = 1.60×10^{-19} C

Mass of electron = 9.11×10^{-31} kg

14B.08.3

Calculate the wavelength of 364 keV gamma ray photons that form 90 % of the emissions.

Planck's Constant, $h = 6.63 \times 10^{-34}$ J s

14B.08.4

The nuclear mass of I-131 is 130.9061246 u

The nuclear mass of Xe-131 is 130.9050824 u

Show that the total energy of the decay from I-131 to Xe-131 is about 970 keV.

1 u = 1.6605×10^{-27} kg

14B.08.5

Write down the decay equation for technetium to ruthenium.

14B.08.6

Consider this interaction:

$$^{111}_{49}\mathrm{In} + {}^{0}_{-1}e^{-} \rightarrow {}^{111}_{48}\mathrm{Cd} + \nu_{\varepsilon}$$

Draw a Feynman diagram to illustrate this event. Show that it is possible.

14B.08.7

Write down the equation that shows the positron decay of fluorine-18.

14B.08.8

A single positron travels about 1 mm before meeting an electron.

A **positronium** particle is briefly formed (a positron and electron).

Explain what happens next.

14B.08.9

Technetium in its metastable state has a half-life of 21700 s.

(a) Calculate the decay constant.

(b) A patient is given a dose of 300 MBq. Calculate the activity after 24 hours.

(c) What proportion of the original dose has decayed?

14B.08.10

The biological half-life of techetium-99m is 86400 s and the physical half-life is 21700 s.

(a) Calculate the effective half-life in s.

(b) Calculate the effective decay constant.

(c) Calculate the activity of the technetium after 3 days, if the initial activity is 3.0×10^6 Bq.

14B.08.11

A radiotherapy machine gives out X-ray photons of 150 keV. A patient needs 1.0 J of energy to treat the cancer cells of a tumour.

(a) Calculate the number of photons required to deliver this dose.

(b) The machine uses a current of 5.0 mA to produce the beam. Assuming that the efficiency of photon production is 1.0 %, and that 5 % of the photons produced are used to irradiate the patient, calculate that time taken for the machine to be switched on.

14B.08.12

A patient who has had a cough for a number of months has seen his doctor. The doctor has done a variety of tests and sent him to the hospital to have a chest X-ray. She has now received the results back from the hospital and she thinks he has a respiratory condition that needs further investigation. There are three imaging techniques that have been suggested:

(a) Ultrasound.

(b) CT scan.

(c) PET scan.

Compare these techniques for convenience, safety, and resolution. Decide which technique should be used in the first instance.

14B.08.13

A sample of radioactive material measured to have an activity of 1200 counts per minute. If the background radiation is 120 counts per minute, calculate:

(a) the activity of the material in Bq.

(b) the number of disintegrations over a 15 hour period.

14B.08.14

What is the absorbed dose if a cancerous tissue of mass 150 g is exposed to 12 J of energy?

14B.08.15

What is the equivalent dose on the tissue in Question 14B.08.14, assuming that neutrons are used for the treatment?

14B.08.16

A technician works with radiation in the form of beta radiation and thermal (slow) neutrons. His monthly dose from the beta radiation is 20 mGy and the thermal neutrons is 50 mGy.

(a) Calculate the total equivalent dose received.

(b) Calculate the daily equivalent dose rate.

Answers to Questions

Tutorial 14B.01

14B.01.1

The eye has:

- a lens
- a photosensitive layer
- a means of focusing
- a connection to the processor to process the images.

The eye is much smaller than a TV camera of similar quality. It is robust, reliable, and self-repairing.

14B.01.2

Low light levels produce poorer visual acuity

because the rods which are sensitive to low light levels are around the yellow spot

and several rods might be connected to one nerve cell.

So, cricketers may not see the small, dark, and fast moving ball.

14B.01.3

Yellow light is the sum of red light and green light.

The sensitivity of the green cones and the red cones add up

to a maximum sensitivity.

14B.01.4

If the aperture of the iris is wide in low light

depth of field is limited.

If the light is bright, the iris has a constricted aperture,

increasing the depth of field.

Tutorial 14B.02

14B.02.1

Power = $1/f$ = 1 ÷ 0.2 D = **5 m**

14B.02.2

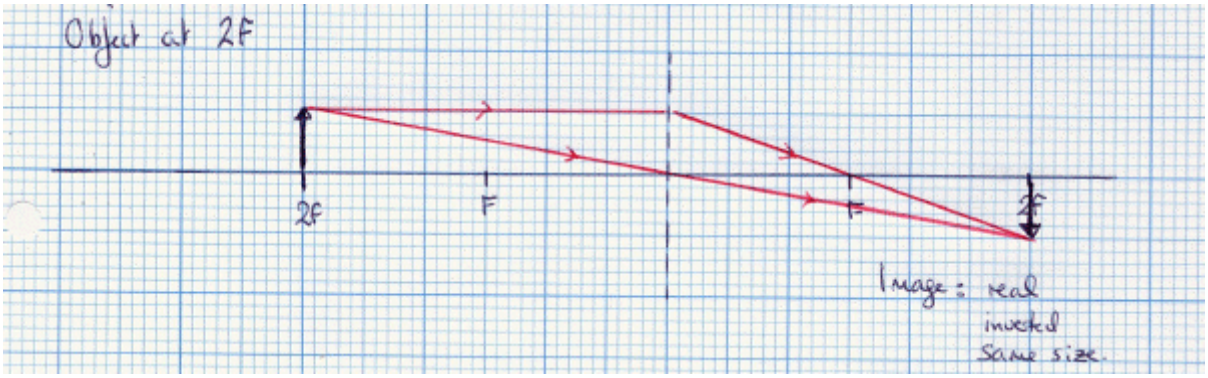


Image is real, inverted, and the same size.

14B.02.3

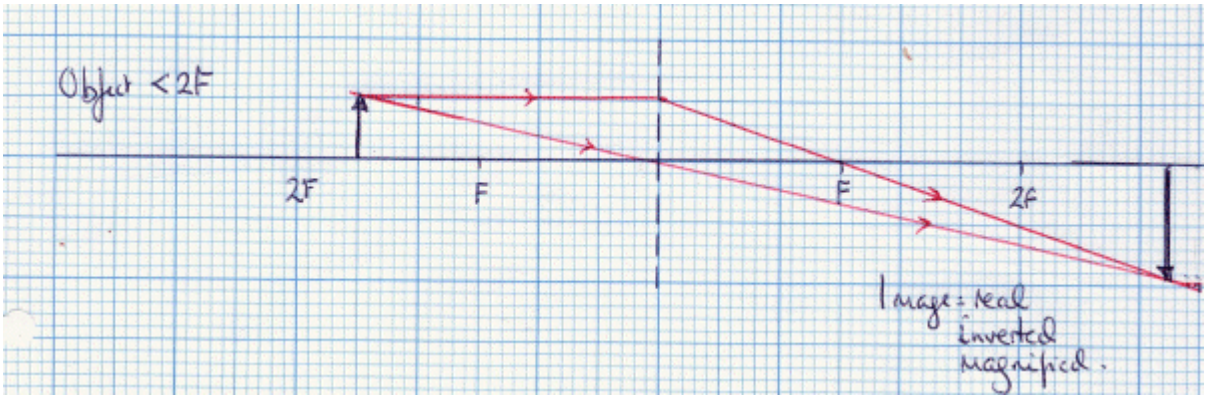


Image is real, inverted, and magnified.

14B.02.4

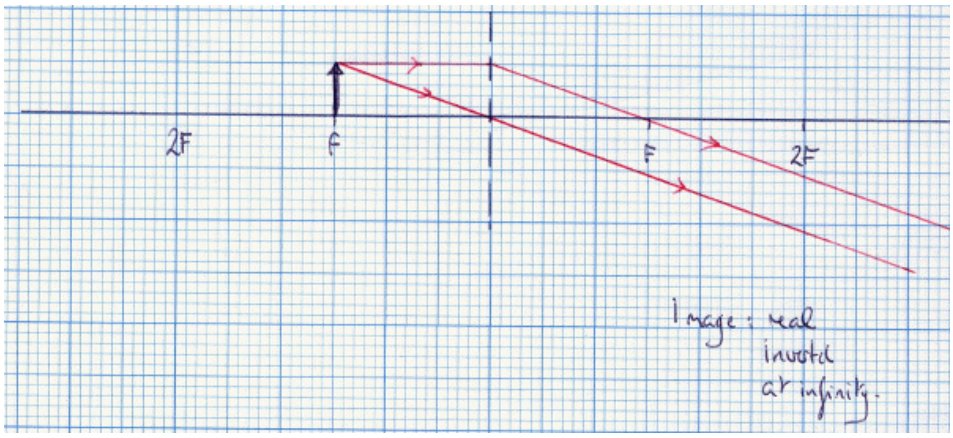


Image is real, inverted, at infinity.

14B.02.5

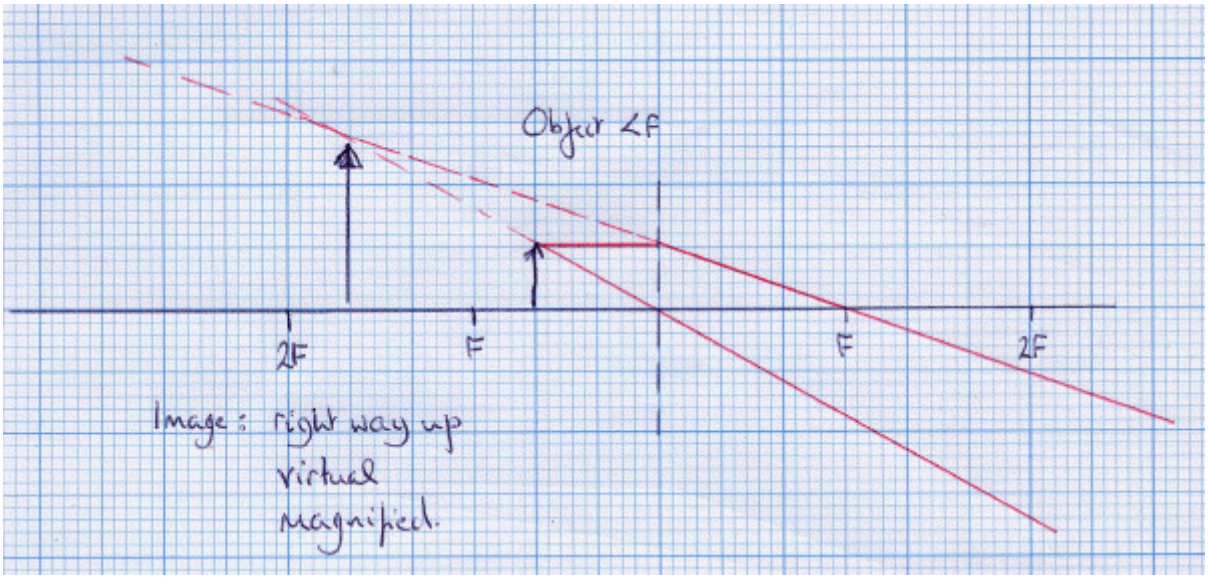


Image is virtual, right way up, and magnified.

14B.02.6

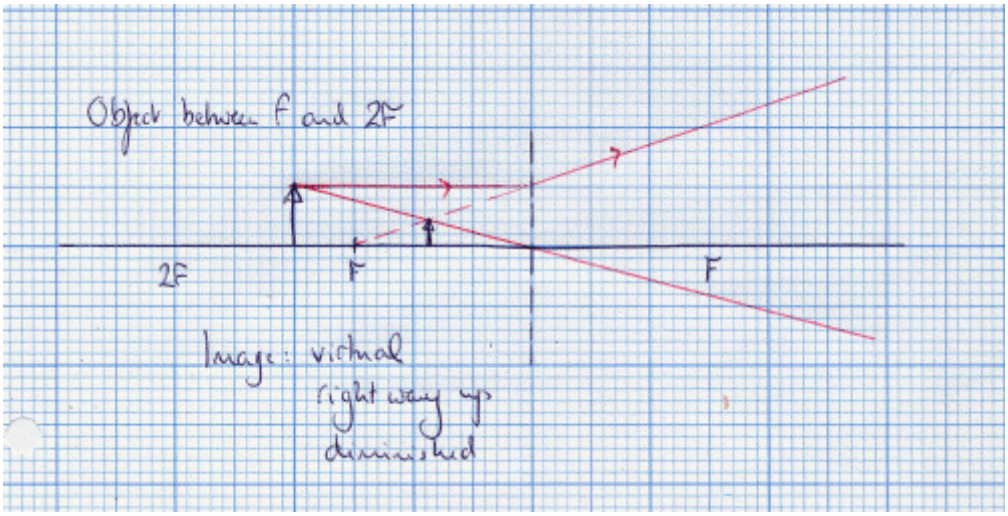


Image is always virtual, right way up, and diminished.

14B.02.7

$$1/f = 1/u + 1/v$$

$1/40 = 1/20 + 1/v$

$1/v = 0.025 - 0.05 = -0.025$

$v =$ **- 40 cm**

magnification = v/u = -40/20 = (-)2

Therefore, the image is **4.4 cm** across

14B.02.8

$$1/f = 1/u + 1/v$$

-1/40 = 1/20 + $1/v$

$1/v$ = -0.025 - 0.05 = -0.075

v = **- 13.3 cm**

magnification = v/u = -13.3/20 = (-)0.667

Therefore, the image is **1.47** cm across

14B.02.9

Power = $1/f$

f = 1/power = 1 ÷ 5 D = **0.2 m**

14B.02.10

Maximum Power of corrected eye = 55 D

Power = $1/u$ + 1/0.02 m

55 = $1/u$ + 1/0.02 m = $1/u$ + 50 D

$1/u$ = 55 D - 50 D = 5.0 D

u = **0.20 m**

14B.02.11

(a)

Power of eye at near point = 1/3.0 m + 1/0.02 m = 0.33 m^{-1} + 50 m^{-1} = **50.33 D**

(b)

Power needed = 1/0.25 m + 1/0.02 m = 4 m^{-1} + 50 m^{-1} = **54 D**

(c)

Power of the corrective lens = Power of the corrected eye - power of the uncorrected eye

Power = 54 D - 50.33 D = **3.67 D**

It's a convex lens as the result is positive.

(d)

Focal length = 1/3.67 D = **0.27 m**

Tutorial 14B.03

14B.03.1

Listener A will detect the sound to the left. The blue wave will have a longer path length than the red.

Listener B will detect the source as being right in front, as both path lengths are equal.

Listener C will detect the source as being to the right as the path length of the red wave is longer than that of the blue.

14B.03.2

Distance = speed × time

Path difference = 340 m s^{-1} × 1.0 × 10^{-5} s = **3.4 × 10^{-3} m** (= 3.4 mm)

14B.03.3

At the open end there is an antinode.

At the closed end there is a node.

This gives half a resonance loop,

which is 1/4 wavelength.

Therefore, the wavelength must be 4 × 2.5 cm = **10 cm**

14B.03.4

The cochlea is like a closed pipe in which the fundamental wavelength is 4 times the length of the pipe.

2.5 cm × 4 = 10 cm = 0.10 m

Use wave equation $c = f\lambda$

$f = c/\lambda$ = 330 m s^{-1} ÷ 0.10 m = **3300 Hz**

14B.03.5

(a)

10^{-12} W m^{-2}. This occurs in the range 1000 to 3500 Hz

(b)

The region of discomfort is at intensities of about 10 W m^{-2} to about 100 W m^{-2}. Pain occurs at any intensity above this. It is not frequency dependent.

(c)

At 10^{-6} W m^{-2} the hearing limits are from about 50 Hz to 15000 Hz.

14B.03.6

30 dB is 10^{-9} W m^{-2}

80 dB is 10^{-4} W m^{-2}

Change in intensity = 10^{-4} W m^{-2} ÷ 10^{-9} W m^{-2} = **100 000**

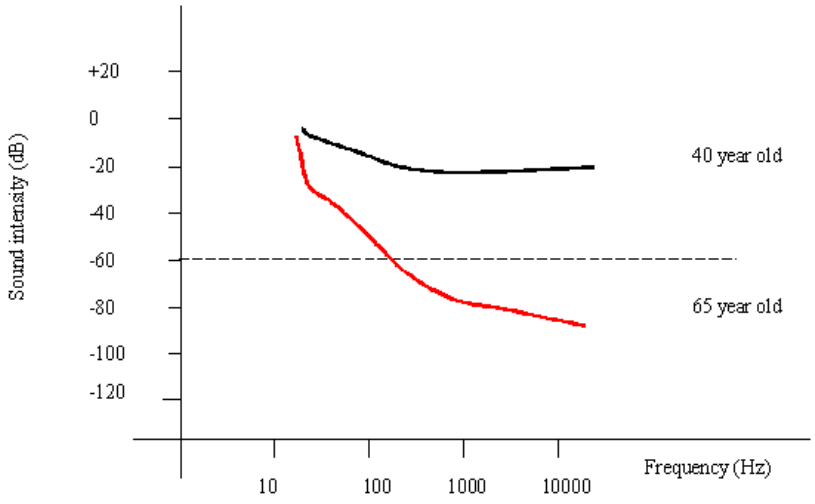
14B.03.7

30 dB is 10^{-9} W m^{-2}

140 dB is 10^{2} W m^{-2}

Change in intensity = 10^{2} W m^{-2} ÷ 10^{-9} W m^{-2}= 10^{11} = **110 dB**

14B.03.8



Tutorial 14B.04

14B.04.1

This is essential because the normal pressure from the heart would rupture the very delicate capillaries in the lung.

14B.04.2

The electrical activity of other nerve cells and muscles could mask the electrical activity of the heart.

14B.04.3

There are high voltages (3000 V at 20 A), which could give a serious shock to another person.

Tutorial 14B.05

14B.05.1

The upper limit of human hearing is 20 000 Hz.

14B.05.2

The forcing frequency is the same as the natural frequency.

14B.05.3

Use $c = f\lambda$

$\lambda = c/f$ = 1500 m s^{-1} ÷ 5.0 × 10^6 Hz = **3.0 ×10^{-4} m** = 0.30 mm

14B.05.4

Find the wavelength of the resonant frequency:

$\lambda = c/f$ = 3791 m s^{-1} ÷ 1.5 × 10^6 Hz = 2.53 × 10^{-3} m = 2.53 mm

Therefore, the thickness of the slice will be half this, **1.27 mm**

14B.05.5

The time taken to reach the boundary is 0.5 × 1.33 × 10^{-4} s = 0.665 × 10^{-4}s

(Remember that the wave is reflected.)

Distance = speed × time = 1450 m s^{-1} × 0.665 × 10^{-4} s **= 0.096 m** (= 9.6 cm)

14B.05.6

Any of these points:

Advantages

- There are no known hazards with low frequency (low energy) beams.
- It is non-invasive.
- There is no discomfort apart from a cold probe!
- More effective than X-ray techniques in producing images of soft tissue.
- The equipment is relatively inexpensive, can be moved about very easily, and does not need a specialist room.
- There are no hazards for the operator.

Disadvantages:

- The sonographer has to be skilled at operating the probe and its associated equipment to get a decent image.
- The image needs skilful interpretation.
- Attenuation can reduce the resolution of the image.
- Bone absorbs ultrasound so that brain images are hard to get.
- Gas-soft tissue interfaces reflect 99.9% of the incident energy. Images of tissues on the far side of lungs are impossible to get.

14B.05.7

Formula:

$$Z = \rho c$$

Z = 1.2 kg m^{-3} × 240 m s^{-1} = **408 kg m^{-2} s^{-1}** ≈ 400 kg m^{-2} s^{-1} (QED)

14B.05.8

Formula:

$$Z = \rho c$$

c = 1.69 × 10^6 kg m^{-2} s^{-1} ÷ 1.06 × 10^3 kg m^{-3} = 1594 m s^{-1} = **1590 m s^{-1}** (3 s.f.)

14B.05.9

Formula:

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

1500 m s^{-1} ÷ 1594 m s^{-1} = sin (2.5 °) ÷ $\sin\theta_2$

$\sin\theta_2$ = 0.0436 ÷ 0.941 = 0.0464

θ_2 = $\sin^{-1}$ (0.0464) = **2.66 °**

The refracted ray will bend away from the normal, because the speed of the sound in the liver is higher than the speed of the sound in water.

14B.05.10

(a) Formula:

$$Z = \rho c$$

For kidney: Z = 1050 kg m^{-3} × 1570 m s^{-1} = **1.65 × 10^{6} kg m^{-2} s^{-1}**

For kidney: Z = 1065 kg m^{-3} × 1590 m s^{-1} = **1.69 × 10^{6} kg m^{-2} s^{-1}**

(b) Formula:

$$\frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

I_r ÷ 260 W m^{-2} = [(1.69 × 10^{6} kg m^{-2} s^{-1} - 1.65 × 10^{6} kg m^{-2} s^{-1}) ÷ (1.69 × 10^{6} kg m^{-2} s^{-1} + 1.65 × 10^{6} kg m^{-2} s^{-1})]^{2}

I_r = [0.040 × 10^{6} kg m^{-2} s^{-1} ÷ 3.34 × 10^{6} kg m^{-2} s^{-1}]^{2} × 260 W m^{-2}

= [0.0120]^{2} × 260 W m^{-2} (Did you remember to square it?)

I_r = **0.0373 W m^{-2}**

14.05.11

(a) Formula:

$$\Gamma = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

Γ = [(429 kg m^{-2} s^{-1} - 2.60 × 10^5 kg m^{-2} s^{-1}) ÷ (429 kg m^{-2} s^{-1} + 2.60 × 10^5 kg m^{-2} s^{-1})]^2

Γ = [-259571 kg m^{-2} s^{-1} ÷ 260429 kg m^{-2} s^{-1}]^2 = [-0.9967]^2

(Did you remember to square it? Note also that the minus sign is got rid of by squaring.)

Γ = 0.993 × 100 % = **99.3 %**

(b)

Almost all the sound will be reflected, so only the outside of the lungs will be shown. The lungs will be opaque to the ultrasound.

14.05.12

(a) Formula:

$$\frac{\Delta f}{f_0} = \frac{2v\cos\theta}{c}$$

Δf = 2.0055 MHz - 2.0000 MHz = 0.0055 MHz = 5500 Hz

(5500 Hz ÷ 2.000 × 10^6 Hz) = ((2v × 0.940) ÷ 1600 m s^{-1})

v = ((5500 Hz ÷ 2.000 × 10^6 Hz) × 1600 m s^{-1}) ÷ 1.879 = 2.341 m s^{-1} = **2.3 m s^{-1}** (2 s.f.)

(b)

No difference will be detected, as cos 90 = 0.

Tutorial 14B.06

14B.06.1

Condition for total internal reflection is that the incident angle for a ray of light is greater than the critical angle.

14B.06.2

The ray would be refracted into the material of higher refractive index.

It does matter as the ray would be lost.

14B.06.3

Use

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$1.55 \sin\theta_c = 1.42 \sin 90$

$\sin\theta_c = 1.42 \div 1.55 = 0.916$

$\theta_c = \sin^{-1}(0.916) =$ **66.4°**

14B.06.4

Angle of refraction = 90° - 66.4° = **23.6°**

14B.06.5

Use

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$1.00 \sin\theta_1 = 1.55 \sin 23.6$

$\sin\theta_1 = 1.55 \times 0.400$

$\theta_c = \sin^{-1}(0.621) =$ **38.4°**

14B.06.6

The uses of an endoscope are:

- diagnosis - the doctor can examine the diseased area.
- taking a biopsy - a small sample of tissue to be tested.
- performing a small operation on the diseased area.
- removing an obstruction.

All of these can be done without having to open the patient up with all the risks that that can bring.

14B.06.7

The partial mirror reflects some of the photons back to stimulate the emission of more photons.

14B.06.8

(a)

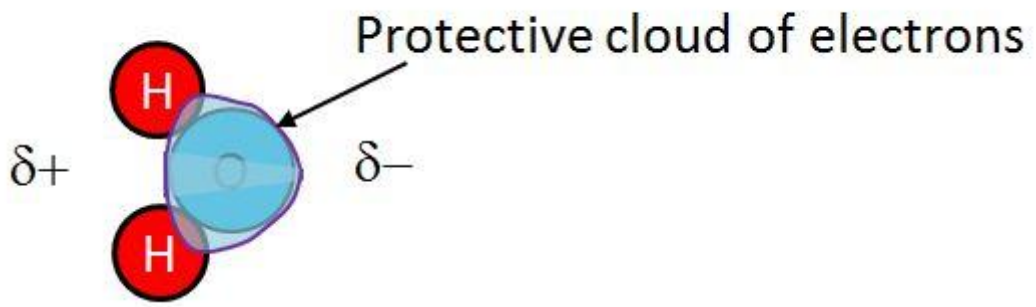
A carbon dioxide laser as its penetration is very small, making it suitable to operate on delicate tissues.

(b)

An argon laser, as the light in the visible range can pass into the eye and causes photocoagulation.

14B.06.9

The electrons form a protective shield around the protons in the oxygen nucleus.



This makes the oxygen atom invisible to the detectors in the MRI scanner.

14B.06.10

The opposing fields will cancel out to give a zero field.

14B.06.11

At a certain frequency, resonance occurs when the forcing frequency is the same as the natural frequency.

The amplitude becomes very large.

(You may remember doing the mass bouncing on a spring experiment using a vibration generator as a driver. Sometimes the amplitude of the oscillations gets so large that the slotted masses detach from the spring completely.)

14B.06.12

Equation:

$$f = 42.6 \times 10^6 B$$

$B = 60 \times 10^6$ Hz $\div$ 42.6×10^6 Hz T^{-1} = **1.4 T**

Tutorial 14B.07

14B.07.1

$$eV = \frac{hc}{\lambda}$$

Rearranging:

$$\lambda = \frac{hc}{eV}$$

Substituting:

$$\lambda = \frac{6.63 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{1.6 \times 10^{-19}\,\text{C} \times V}$$

Evaluating:

$$\lambda = \frac{1.24 \times 10^{-6}\,\text{J m C}^{-1}}{V}$$

This is sometimes called the Duane-Hunt Law.

14B.07.2

Use

$$\lambda = \frac{1.24 \times 10^{-6}\,\text{J m C}^{-1}}{V}$$

$\lambda = 1.24 \times 10^{-6}\ \text{J m C}^{-1} \div 50\,000\ \text{V}$

$\lambda =$ **2.48×10^{-11} m.**

14B.07.3

(a)

Power = VI = 100 000 V × 0.04 A = **4000 W**

(b)

Number of electrons per sec = 0.04 A ÷ 1.6 × 10^{-19} C = **2.5 × 10^{17} s^{-1}**

(c)

Number of photons is about 1/100 of this, **2.5 × 10^{15} s^{-1}**

(d)

Photon energy = 100 000 V × 1.6 × 10^{-19} C = 1.6 x 10^{-14} J

Wavelength = hc/E = (6.63 × 10^{-34} J s × 3.0 × 10^{8} m s^{-1}) ÷ 1.6 × 10^{-14} J

= **1.24 × 10^{-11} m** = 0.0124 nm

14B.07.4

Power is 200 000 V × 0.025 A = 5000 W

99% is heat.

Therefore heating effect is 4950 J s^{-1}

Anode has a mass of 1 kg.

Temperature change = 4950 J s^{-1} ÷ (300 J $kg^{-1}K^{-1}$ × 1 kg) = 16.5 K s^{-1}

Temperature rise to melting point = 3000 K - 300 K = 2700 K

Time taken to get to melting point = 2700 K ÷ 16.5 K s^{-1} = 164 s

The anode will have melted. Oh dear, that's expensive!

14B.07.5

99.9% of ultrasound waves are reflected at the lung-air boundary.

There is so little transmission...

...and even less reflection at the other side so that getting images is impossible.

14B.07.6

μ_m for copper = 693 m^{-1} ÷ 8930 kg m^{-3} = **0.0776 m^2 kg^{-1}**

14B.07.7

μ for Aluminium = 0.693 ÷ 3.2 x 10^{-3} m = **217 m^{-1}**

14B.07.8

Power of the X-ray tube = 80 000 V × 0.050 A = 4000 W

The Power converted to X-rays = 4000 W × 0.01 = 40 W

Intensity = 40 W ÷ (10 × 10^{-6} m^2) = **4 × 10^6 W m^{-2}**

14B.07.9

Use the inverse square law:

Distance goes from 0.01 m to 1.5 m which is 150 times

Intensity reduces by 150^2 = 22500

Intensity now is 4 × 10^6 W m^{-2} ÷ 22500 = **178 W m^{-2}**

14B.07.10

Use

$$I = I_0 e^{-\mu x}$$

I = 177 x $e^{-217 \times 0.005}$ = 177 x $e^{-1.085}$ = 177 x 0.338

I = **60 W m^{-2}**

14B.07.11

The photomultiplier tube is in the X-ray beam,

which would expose the viewer to a high dose of X-rays.

In particular, the head would be in the beam, which is not a good idea.

X-rays are also not very good for the eyes.

Tutorial 14B.08

14B.08.1

Alpha particles have a very limited range, being stopped by a sheet of paper. They would be stopped by a few layers of cells, so would not get out of the body. The range is about 37 mm.

The alpha particles are intensely ionising and would do a lot of damage to the DNA of the cells surrounding the radioisotope. Short half-life emitters would be particularly destructive, as they give out their radiation over a short period of time. The damage is also likely to be so serious that radiation sickness will ensue.

Additionally, the alpha emitters are heavy metals which can be chemically toxic to the body.

Not a good idea.

14B.08.2

Convert eV to J:

$$3.34 \times 10^{5}\ \text{eV} \times 1.60 \times 10^{-19}\ \text{J eV}^{-1} = 5.34 \times 10^{-14}\ \text{J}$$

Use:

$$v^2 = \frac{2E_k}{m}$$

$$v^2 = (2 \times 5.34 \times 10^{-14}\ \text{J}) \div 9.11 \times 10^{-31}\ \text{kg} = 1.17 \times 10^{17}\ \text{m}^2\ \text{s}^2$$

$$v = \mathbf{3.42 \times 10^{8}\ m\ s^{-1}}$$

The beta minus particle cannot travel at this speed, since it's above the speed of light. Some energy must have passed to the electron antineutrino. The electron, as it approaches the speed of light, will gain mass due to relativity. This will allow the electron to more energy than the 256 keV it would have at the speed of light.

14B.08.3

Convert eV to J:

3.64×10^5 eV $\times$ 1.60×10^{-19} J eV^{-1} = 5.82×10^{-14} J

Use:

$$\lambda = \frac{hc}{E}$$

$\lambda = (6.63 \times 10^{-34}$ J s $\times$ 3.0×10^8 m s$^{-1}) \div 5.82 \times 10^{-14}$ J = **3.42×10^{-12} m**

14B.08.4

Calculate the mass change:

Δm = 130.9061246 u - 130.9050824 u = 0.0010422 u

Convert this into kilograms:

Δm = 0.0010422 u $\times$ 1.6605×10^{-27} kg u^{-1} = $1.7305731 \times 10^{-30}$ kg

Use $E = mc^2$ to calculate the energy:

E = $1.7305731 \times 10^{-30}$ kg $\times$ $(3.00 \times 10^8$ m s$^{-1})^2$ = 1.5575×10^{-13} J

Convert this to eV:

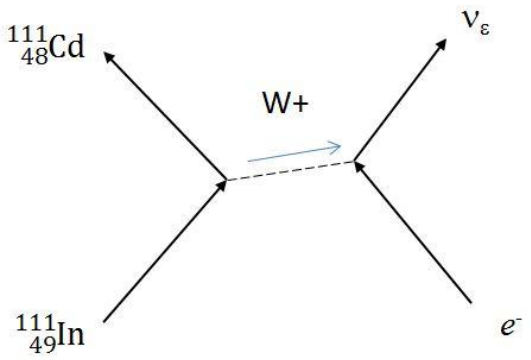
E = 1.5575×10^{-13} J $\div$ 1.60×10^{-19} J eV^{-1} = **973×10^3 eV** = 970 keV (QED)

14B.08.5

$$^{99}_{43}\mathrm{Tc} \rightarrow {}^{99}_{44}\mathrm{Ru} + {}^{0}_{-1}e^{-} + \bar{\nu}_{\varepsilon}$$

14B.08.6

Diagram



Conservation Rules

$$In + e^- = Cd + \nu_e$$

Q: 49 + -1 = 48 + 0 (Yes)

B: 111 + 0 = 111 + 0 (Yes)

Le: 0 + +1 = 0 + +1 (Yes)

Note that the baryon number for the nuclides is the same as the nucleon number. There are no strange quarks, so S is ignored

14B.08.7

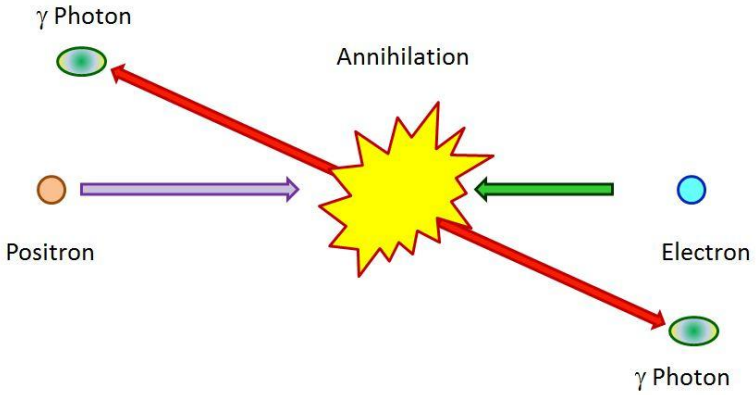
$$^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{+1}\beta^{+} + \nu_\varepsilon$$

14B.08.8

The positron and the electron annihilate...

...to release two gamma photons.

The energy of the gamma photons is 511 keV.



14B.08.9

(a) Use:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

$\lambda = 0.693 \div 21700\ s = \mathbf{3.194 \times 10^{-5}\ s^{-1}}$

(b) Use:

$$A = A_0 e^{-\lambda t}$$

24 h = 86400 s

$A = 300 \times 10^6\ Bq \times e^{-3.194 \times 10\text{^-}5\ s\ \text{^-}1 \times 86400\ s}$

$A = 300 \times 10^6\ Bq \times e^{-2.760}$

$A = 300 \times 10^6\ Bq \times 0.0633 = \mathbf{19.0 \times 10^6\ Bq} = 19\ MBq$

(c)

$300 \times 10^6\ Bq - 19.0 \times 10^6\ Bq = 281 \times 10^6\ Bq$

Proportion decayed = $(281 \div 300) \times 100\ \% = 94\ \%$

14B.08.10

(a) Use:

$$\frac{1}{t_{\frac{1}{2}\mathrm{E}}} = \frac{1}{t_{\frac{1}{2}\mathrm{P}}} + \frac{1}{t_{\frac{1}{2}\mathrm{B}}}$$

$t_{1/2E}^{-1}$ = (21700 s)$^{-1}$ + (86400 s)$^{-1}$ = 57.66 × 10^{-6} s^{-1}

$t_{1/2E}$ = **17344 s**

(b) Use:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

λ = 0.693 ÷ 17344 s = **3.996 × 10^{-5} s^{-1}**

(c) Use:

$$A = A_0 e^{-\lambda t}$$

24 h = 86400 s

A = 300 × 10^{6} Bq * e$^{-3.996 \times 10\text{^-}5\ s\ \text{^-}1 \times 3\ d \times 86400\ s\ d\text{^-}1}$

A = 300 × 10^{6} Bq × e$^{-10.35}$

A = 300 × 10^{6} Bq × 3.17 × 10^{-5} = **9.5 × 10^{3} Bq**

14B.08.11

(a)

Photon energy = 1.50×10^{5} eV × 1.60×10^{-19} J eV^{-1} = 2.4×10^{-14} J

1.0 J contains **4.17×10^{13}** photons.

(b)

1.0 A = $(1.602 \times 10^{-19}\ C)^{-1}$ = 6.24×10^{18} electrons per second.

Current = 5.0×10^{-3} A × 6.24×10^{18} electrons per second = $3.12 \times 10^{16}\ s^{-1}$

Number of photons per second = $3.12 \times 10^{16}\ s^{-1} \times 0.01$ = **$3.12 \times 10^{14}\ s^{-1}$**

Since 5 % of these are used,

the useful number of photons per second = $3.12 \times 10^{14}\ s^{-1} \times 0.05 = 1.56 \times 10^{13}\ s^{-1}$

Time taken = 4.17×10^{13} photon ÷ $1.56 \times 10^{13}\ s^{-1}$ photons s^{-1} = **2.67 s**

14B.08.12

(a)

Ultrasound is safe to use, as it is a non-ionising radiation. It is convenient for the patient no preparation is needed, beyond removing clothing. Cold coupling gel may be uncomfortable for a couple of seconds. However, the sonographer has to be skilled in interpretation of the images. The resolution is about 1 mm. The sonographer is seated next to the patient, so that there is personal contact.

However, for this investigation, it would be quite useless, because the lungs are full of air. Therefore, the ultrasound will be almost completely reflected by the lung tissue, so the ultrasound waves will not penetrate into the lung space.

(b)

A CT scan involves X-rays which are ionising radiation. There are risks, but these are minimised by ensuring the dose is as low as reasonably achievable (ALARA). The radiographers always operate the machine in a separate room, having contact with the patient through a window and an intercom. The patient needs to keep still during the procedure. He will also need to be injected with a dye to enhance the contrast. This can have side-effects.

The CT scanner takes images of the area of the body under investigation from different angles. The head goes around the patient on a vertically rotating gantry. The images are collated by the computer, which can generate "slices" through the body. The resolution can be 100 micrometres. It can also make a 3-dimensional image for the radiologist to interpret. The images that the computer produces show a picture of the tissue, and the doctor can decide whether there are abnormalities.

(c)

A PET scan uses a tracer that emits positrons. The tracer is a short lived isotope of fluorine that has too few neutrons for stability. The patient has to be prepared by injection of the radiopharmaceutical which has to be made up soon before the procedure.

When the emitted positrons meet an electron, an annihilation event occurs, sending out two gamma photons in opposite directions. These are detected by gamma cameras that are mounted on a gantry around the patient. The images are not easy to interpret, but computers produce an image that a skilled radiologist can interpret. Resolution is not that good, about 10 mm, as the gamma photons can blur the image. It can be improved by combining the PET scan with a CT scan.

Gamma radiation is highly penetrating, and care has to be taken to reduce the exposure of the patient to as low as possible. The radiographers have to work in a separate room. The wall between the two rooms will be lead-lined to attenuate the gamma radiation. The radiographers will wear film badges to monitor their exposure to radiation.

14B.08.13

(a)

Activity of the sample = 1200 min^{-1} - 120 min^{-1} = 1080 min^{-1}.

1080 ÷ 60 = **18 Bq**

(b)

15 h = 54 000 s

Number of disintegrations = 54000 s × 18 Bq = **972 000** disintegrations

14B.08.14

Dose = energy ÷ mass = 12 J ÷ 0.150 kg = 80 Gy

14B.08.15

Dose = energy ÷ mass = 12 J ÷ 0.150 kg = 80 Gy

Equivalent dose = 80 Gy × 3 = **240 Gy**

14B.08.16

(a)

Dose from gamma rays = 20×10^{-3} Gy × 1 = 20×10^{-3} Sv

Dose from neutrons = 50×10^{-6} Gy × 3 = 150×10^{-6} Sv

Total = **2.015×10^{-3} Sv.**

(b)

Daily dose = 2.105×10^{-3} Sv ÷ 30 = 6.72×10^{-4} Sv = **0.67 mSv day^{-1}**